

Problem: η is a chain map but

Question: When is η a homology isom?

Need homotopy pullbacks:

Recall replacement of maps $f: Y \rightarrow X$

by fibrations $\pi: E_f \rightarrow X$:

$$E_f = \{ (y, \omega) \mid y \in Y, \omega: I \rightarrow X, f \circ \omega = \omega(0) \}$$

$$Y \xrightarrow{f} X \quad : \quad \pi(y, \omega) = \omega(1)$$

$$i(y) = (y, \omega_y)$$

$$E_f$$

given pullback $Y \times_X Z \rightarrow Z$

$$\begin{array}{ccc} & & \downarrow g \\ Y & \xrightarrow{f} & X \\ & & \downarrow \end{array}$$

homotopy pullback is

$$\begin{array}{ccc} E_f \times_X E_g & \rightarrow & E_g \\ \downarrow & & \downarrow \\ E_f & \rightarrow & X \end{array}$$

Note map: $Y \times_X Z \rightarrow E_f \times_X E_g$

Notation: homotopy pullback = $E_f \times_X E_g = Y \times_X Z$