Some good properties:

a) <u>balanced</u>: g fibration $\Rightarrow$

$$Y \times_X Z \xrightarrow{\simeq} E_f \times_X E_g$$

Cor: $Y \times_X E_g \xrightarrow{\simeq} E_f \times_X E_g$

b) <u>homotopy invariance of homotopy pullbacks</u>

given a map which is a homotopy eq on Y, Z, X corners:

$$\begin{array}{ccc} Z & & Z_1 \\ \downarrow & \simeq & \downarrow \\ Y \to X & & Y_1 \to X_1 \end{array}$$

i.e.

$$\Rightarrow Y \overset{\sim}{\times}_X Z \xrightarrow{\simeq} Y_1 \times_{X_1} Z_1 \text{ is a h. eq.}$$

But:

$$\begin{array}{ccc} * & & PX \\ \downarrow & \simeq & \downarrow \\ * \to X & & * \to X \end{array} \qquad \not\Rightarrow$$

$$* = * \times_X * \xrightarrow{\simeq} * \times_X PX = \Omega X.$$