

The comodule analogue of homotopy pullbacks is completed by the following:

6

- 1) Cartan constructions are the analogue of fibrations
- 2) Cartan constructions replace comodules by assembling proper injective resolutions
- 3) Make a chain model which is balanced and homologically invariant
- 4) Prove that the Eilenberg-Moore construction gives a chain model for a homotopy pullback.

### Cartan constructions:

$C =$  simply connected differential coalgebra which is free over  $R =$  base

$F =$  chain complex over  $R =$  fibre

$E =$  construction over  $C =$  total space:

a)  $E = F \otimes C$  as a comodule, i.e.

$$\Delta: E = F \otimes C \xrightarrow{1 \otimes \Delta} F \otimes C \otimes C = E \otimes C$$

is coaction

b)  $E$  is a differential comodule over  $C$

and  $F \equiv F \otimes R \subseteq E$  is a subcomplex.

Proposition: b) is equivalent to:

$\exists$  linear maps of degree  $-1$ ,

$$\tau: F \otimes C \rightarrow F \text{ such that}$$