The chain model analogous to a homotopy pullback:

$$M \,\tilde{\square}_C\, N = E(M) \,\square_C\, E(N)$$

it is balanced: M a construction $\Longrightarrow$

$N \to N_1$ a homology isom

$$M \,\square_C\, N \xrightarrow{\cong} M \,\square_C\, N_1$$

homology isom

Proof: write $M = F \otimes C$

$$\begin{array}{ccc} M \,\square_C\, N & \longrightarrow & M \,\square_C\, N_1 \\ \| & & \| \\ F \otimes C \,\square_C\, N & & F \otimes C \,\square_C\, N_1 \\ \| & & \| \\ F \otimes N & \longrightarrow & F \otimes N_1 \end{array}$$

$$N \xrightarrow{\cong} N_1 \Longrightarrow F \otimes N \xrightarrow{\cong} F \otimes N_1$$

Cor: $M \,\square_C\, E(N) \xrightarrow{\cong} E(M) \,\square_C\, E(N) = M \,\tilde{\square}_C\, N$

weak homological invariance:

$$N \xrightarrow{\cong} N_1 \Longrightarrow M \,\tilde{\square}_C\, N \longrightarrow M \,\tilde{\square}_C\, N_1$$

Proof: $E(M) = F \otimes C$, $N \xrightarrow{\cong} N_1 \Longrightarrow$

$$M \,\tilde{\square}_C\, N = E(M) \,\square_C\, N = F \otimes N \xrightarrow{\cong} F \otimes N_1 = E(M) \,\square_C\, N_1 = M \,\square_C\, N_1$$