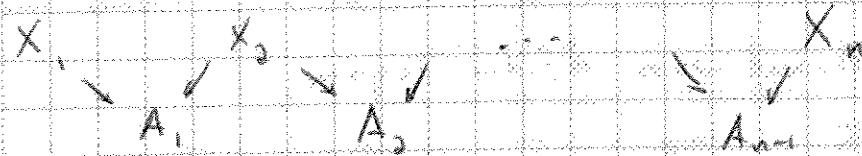
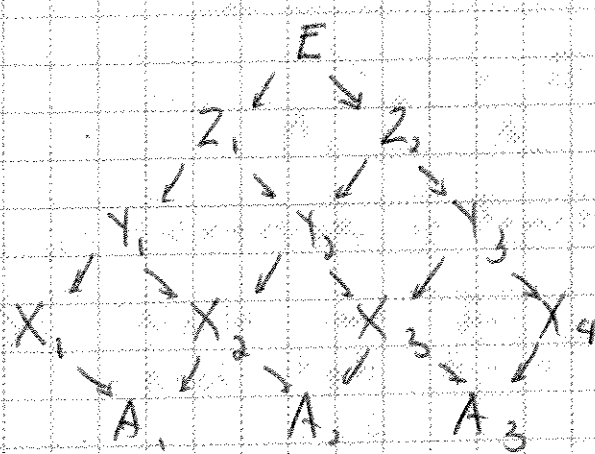


# Lecture 2. Iterated pullbacks and loop multiplication

Data for an iterated pullback is a zigzag diagram



which expands to pullback squares, e.g.  $n=4$ :



$$\begin{aligned}
 E &= X_1 \times_{A_1} X_2 \times_{A_2} X_3 \times_{A_3} X_4 \\
 &= \text{iterated pullback}
 \end{aligned}$$

## Construction of the iterated homotopy pullback:

Convert all  $X_1 \rightarrow A_1, X_2 \rightarrow A_1 \times A_2, \dots, X_n \rightarrow A_n$   
 to fibrations  $E_1 \rightarrow A_1, E_2 \rightarrow A_1 \times A_2, \dots, E_n \rightarrow A_n$   
 (Note:  $\therefore$  all  $E_i \rightarrow A_i$  are fibrations)

There is a map:

$$X_1 \times_{A_1} X_2 \times_{A_2} \dots \times_{A_n} X_n \rightarrow E_1 \times_{A_1} E_2 \times_{A_2} \dots \times_{A_n} E_n$$

|| def

iterated homotopy pullback

|| notation

$$X_1 \tilde{\times}_{A_1} X_2 \tilde{\times}_{A_2} \dots \tilde{\times}_{A_n} X_n$$