the differential in $\Omega C \otimes R \otimes \Omega C$ is the tensor product differential, and the differential in $\Omega C \otimes_\tau C \otimes_\tau \Omega C$ is $d_{\tau,\tau} =$

$d_\tau \otimes 1 + 1 \otimes d_\tau - 1 \otimes d_C \otimes 1$.

(The last statement needs some justification: the identification $C \xrightarrow{\cong} C \,\square_C\, C \,\square_C\, C$ makes $d_C \equiv d_C \otimes 1 \otimes 1 + 1 \otimes d_C \otimes 1 + 1 \otimes 1 \otimes d_C$. The above formula for $d_{\tau,\tau}$ has the correct number of $1 \otimes d_C \otimes 1$ terms.)

We close with 3 related easy facts:

1) the map $\Theta : \Omega C \otimes_\tau C \otimes_\tau \Omega C \to \Omega C$ given by $\begin{cases} \Theta(\alpha \otimes 1 \otimes \beta) = \alpha\beta \\ \Theta(\alpha \otimes c \otimes \beta) = 0, \ c \in \overline{C} \end{cases}$ is a <u>chain map</u>.

2) Since $C \otimes_\tau \Omega C$ is a construction and $R \to \Omega C \otimes_\tau C$ is a homology isom, the map

$$\Omega C = R \,\square_C\, C \otimes_\tau \Omega C \to \Omega C \otimes_\tau C \,\square_C\, C \otimes_\tau \Omega C$$
$$= \Omega C \otimes_\tau C \otimes_\tau \Omega C$$

is a homology isom.

3) $\therefore\ \Omega C \otimes_\tau C \otimes_\tau \Omega C \xrightarrow{\Theta} \Omega C$ is a homol isom.

Hence, the multiplication in $\mathrm{Cotor}^C(R,R)$ is induced by

$$\Omega C \otimes \Omega C = \Omega C \otimes R \otimes \Omega C \xrightarrow{\simeq} \Omega C \otimes_\tau C \otimes_\tau \Omega C \xrightarrow{\Theta} \Omega C$$

which is exactly the multiplication in ΩC.