

Proof for  $n \geq 3$ !

$$X_1 \times_{A_1} X_2 \xrightarrow{\cong} T_1 \square_{A_1} T_2 \text{ by } n=2 \text{ case.}$$

$T_2$  a b) construction  $\Rightarrow T_1 \square_{A_1} T_2$  is a construction over  $A_2 \Rightarrow$

$$E = X_1 \times_{A_1} X_2 \times_{A_2} X_3 = (X_1 \times_{A_1} X_2) \times_{A_2} X_3$$

$$\cong (T_1 \square_{A_1} T_2) \square_{A_2} T_3$$

is a homology isom.

Note: The augmentation ~~is~~ homology isomorphism is a composition

$$E \xrightarrow{\cong} E \square_{A_1} E \square_{A_2} \dots \square_{A_{n-1}} E$$

$$\cong X_1 \square_{A_1} X_2 \square_{A_2} \dots \square_{A_{n-1}} X_n$$

$$\cong T_1 \square_{A_1} T_2 \square_{A_2} \dots \square_{A_{n-1}} T_n$$

That is,  $HE \cong \text{Cotor}_{A_1 \rightarrow \dots \rightarrow A_{n-1}}(X_1, \dots, X_n)$  is an isomorphism of functors.