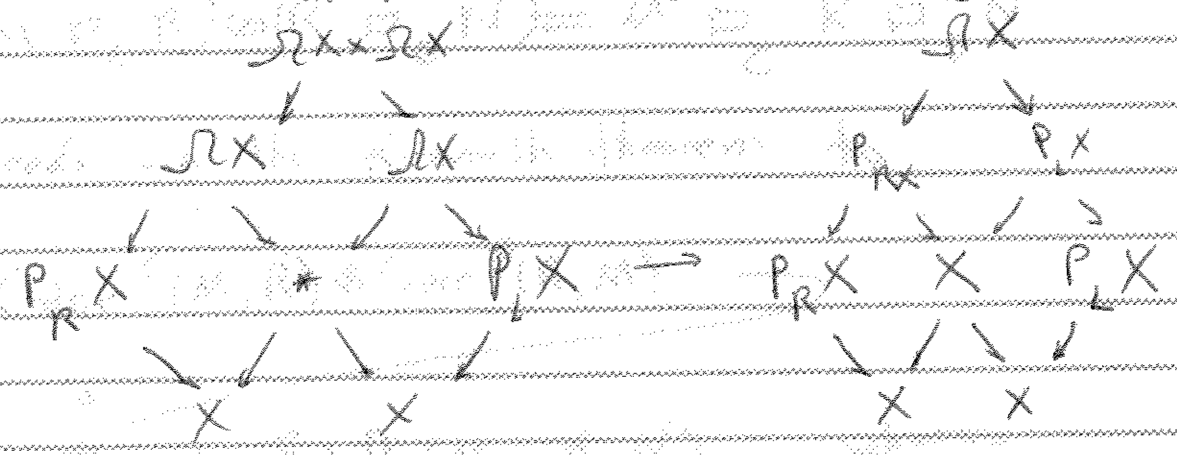
Loop multiplication is given by the map of iterated homotopy pullback diagrams:



where: $P_R X = \{ \omega : I \to X \mid \omega(0) = * \}$

$P_L X = \{ \omega : I \to X \mid \omega(1) = * \}$

$\tilde{\Omega} X = \{ (\omega, \gamma) \in P_R X \times P_L X \mid \omega(1) = \gamma(0) \} \xrightarrow{\simeq} \Omega X$

via: $(\omega, \gamma) \longmapsto \omega * \gamma$

i.e. $\Omega X \times \Omega X \xrightarrow{\mu} \tilde{\Omega} X$

∴ We have the differential cotor version of loop multiplication

$$H(\Omega X \times \Omega X) \xrightarrow{\mu_*} H(\tilde{\Omega} X)$$

$$\| \qquad\qquad\qquad\qquad \|$$

$$\mathrm{Cotor}^{X,X}(P_R X, R, P_L X) \longrightarrow \mathrm{Cotor}^{X,X}(P_R X, X, P_L X)$$

The remainder of this lecture is devoted to discussing the map μ and relating it to the cobar or loop construction ΩC.

Discussing μ involves splitting and collapsing differential Cotor.