$\therefore$ $\mathrm{Cotor}^C(R,R)$ is an algebra via:

$$\mathrm{Cotor}^C(R,R) \otimes \mathrm{Cotor}^C(R,R) \longrightarrow \mathrm{Cotor}^{C,C}(R,R,R) \longrightarrow \mathrm{Cotor}^{C,C}(R,C,R) \longrightarrow \mathrm{Cotor}^C(R,R)$$

Fact: This is an associative multiplication.

Geometrically, it is

$$H(\Omega X) \otimes H(\Omega X) \to H(\Omega X \otimes \Omega X) \to H(\Omega X \times \Omega X) \xrightarrow{\mu} H(\tilde{\Omega} X).$$

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Relation to the multiplication in the cobar construction:

$C$ = simply connected differential coalgebra

$\Omega C = T(s^{-1}\overline{C})$ = tensor algebra with differential a derivation given on generators by

$$d s^{-1}c = -s\,dc + \sum (-1)^{\deg c'} s^{-1}c' \otimes s^{-1}c''$$

where $\overline{\Delta}(c) = \sum c' \otimes c''$.