

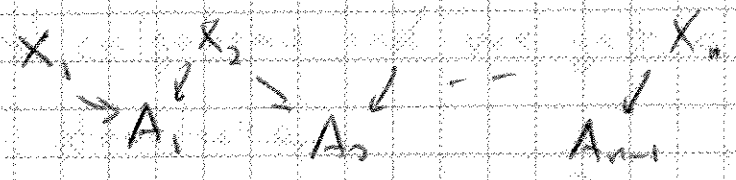
Lecture 3. Computation of the mod p homology of $\Omega^{2g+2n+1}$, $p = \text{an odd prime}$

Coproducts in differential Cotor

1) coproducts in $\text{Cotor}^{A_1, \dots, A_n}(X_1, \dots, X_n)$ exist because of the fact that the Eilenberg-Zilber map is a natural transformation of differential coalgebras

$$\nabla: X \otimes Y \rightarrow X \times Y$$

∴ These coproducts are compatible with the Eilenberg-Moore chain models. In detail, suppose E is the pullback of the homotopy pullback diagram



and assume HE is torsion free and A_1, \dots, A_n are all \mathbb{Z} -reduced simplicial sets. Write $\text{Cotor}^A(X) = \text{Cotor}^{A_1, \dots, A_n}(X_1, \dots, X_n)$. Then

the compatibility of the coproducts is expressed by

$$\begin{array}{ccccccc} HE & \xrightarrow{\Delta} & H(E \times E) & \xleftarrow{\nabla} & H(E \otimes E) & \xleftarrow{\Delta} & HE \otimes HE \\ \downarrow \cong & & \downarrow \cong & & \downarrow \cong & & \downarrow \cong \\ \text{Cotor}^A(X) & \xrightarrow{\Delta} & \text{Cotor}^{A \times A}(X \times X) & \xleftarrow{\nabla} & \text{Cotor}^{A \otimes A}(X \otimes X) & \xleftarrow{\Delta} & \text{Cotor}^A(X) \otimes \text{Cotor}^A(X) \end{array}$$