Let $F \xrightarrow{i} E \xrightarrow{\pi} B$ be a fibration with E acyclic. The suspension is
$$\sigma : \bar{H}_{k-1} F \xleftarrow[\cong]{\partial} H_k(E, F) \xrightarrow{\pi} \bar{H}_k(B).$$

In terms of the Eilenberg-Moore chain models, let $E = C \otimes F$ be an acyclic construction. The suspension is
$$\sigma : \bar{H}_{k-1} F \xleftarrow[\cong]{\partial} H_k(E, F) \longrightarrow \bar{H}_k(C)$$
and, in the computation above, it is clear that $\sigma x_{2np^k-1} = z^{p^k}$ where x_{2np^k-1} is the exterior generator in dimension $2np^k-1$.

(This is just another way of saying that $d(z^{p^k} \otimes 1) = 1 \otimes x_{2np^k-1}$ in the acyclic construction.)