2) in general, $\mathrm{Cotor}^C(M,N)$ or even $\mathrm{Cotor}^C(R,R)$ <u>may have no coalgebra structure</u>. Even if it does, it <u>may have nothing to do with geometry</u>.

2a) for example, $H(\Omega\Sigma X) = T(\bar{H}X)$
$$= \mathrm{Cotor}^{\Sigma X}(R,R) = \mathrm{Cotor}^{H(\Sigma X)}(R,R),$$
$\bar{H}X$ torsion free has a variety of coalgebra structures depending on $\bar{\Delta} : \bar{H}X \to \bar{H}X \otimes \bar{H}X$ even though $H(\Sigma X)$ has a trivial coalgebra structure

2b) if C has a commutative diagonal, then $\Delta : C \to C \otimes C$ is a map of differential coalgebras and we get a coalgebra structure
$$\begin{aligned} \mathrm{Cotor}^C(R,R) \to \mathrm{Cotor}&^{C\otimes C}(R,R) \\ &\| \\ &\mathrm{Cotor}^C(R,R) \otimes \mathrm{Cotor}^C(R,R) \end{aligned}$$
when $\mathrm{Cotor}^C(R,R)$ is torsion free.

3) In the case of $\mathrm{Cotor}^C(R,R)$ or $\mathrm{Cotor}^A(R,R)$, the coalgebra structures in 1) or 2) are compatible with the products in the previous lectures, that is, <u>these are</u>