

The homology algebra $H(\Omega S^{2n+1})$ with \mathbb{Z}/p - coefficients, $p =$ odd prime:

Step 1. Let R be any comm ring. Then

$H(\Omega S^{2n+1}) = P[\mathbb{Z}_{2n}] =$ polynomial algebra

and ΩS^{2n+1} is formal, that is, there is

a homology isom of differential coalgebras

$$\Theta: P[\mathbb{Z}_{2n}] \rightarrow C(\Omega S^{2n+1}) = \Omega S^{2n+1}$$

Proof: Use the associative Moore loop space.

Then $C(\Omega S^{2n+1}) = \Omega S^{2n+1}$ is an associative

differential algebra. The $2n$ -th Eilenberg

subcomplex of ΩS^{2n+1} is homotopy equivalent

and has simplices with $2n$ -skeleton = pt.

\therefore \exists primitive cycle $x_{2n} \in C_{2n}(\Omega S^{2n+1})$

which represents a homology generator.

Set $\Theta \left(\begin{smallmatrix} k \\ \mathbb{Z}_{2n} \end{smallmatrix} \right) = x_{2n}^k$ and check

that Θ is a ~~map~~ homology isom of

d.f.d coalgebras.