<u>Step 3.</u> Let $A_k = \langle 1, z, \ldots, z^{p-1} \rangle$, $z^p = 0$,

$|z| = 2p^k n$

A_k = commutative algebra $\Rightarrow$

$\mathrm{Tor}_{A_k}(R, R)$ = Hopf algebra with commutative multiplication $\forall R$.

Claim: $\mathrm{Tor}_{A_n}(R, R) = \Gamma(t) \otimes E(w)$
as Hopf algebras
where: $|w| = |z| - 1$, $|t| = p|z| - 2$,

$E(w) = \langle 1, w \rangle$ = ext alg, $\Delta(w) = w \otimes 1 + 1 \otimes w$

$\Gamma(t) = \langle 1, t = \gamma_1, \gamma_2, \ldots \rangle$ = divided power alg, $|\gamma_i| = i|t|$, $\gamma_i \cdot \gamma_j = \binom{i}{j} \gamma_{i+j}$.

Proof: Consider $B = \Gamma(t) \otimes E(w) \otimes A_k$

where: $dz = 0$, $dw = z$, $dt = wz^{p-1}$,

$d\gamma_k = \gamma_{k-1}\, dt$

Claim: B is an acyclic DGA.