

$$\begin{aligned} \therefore \text{Tor}_{A_R}(R, R) &= H(B \otimes_{A_R} R) \\ &= B \otimes_{A_R} R = \Gamma(t) \otimes E(w) \end{aligned}$$

as algebras.

Let  $R = \mathbb{Z}$ . Then  $\Delta(t) = t \otimes 1 + 1 \otimes t$ ,

$$t^i = i! \gamma_i \Rightarrow$$

$$\Delta(\gamma_i) = \frac{1}{i!} [\Delta(t)]^i = \frac{1}{i!} \sum_{\alpha+\beta=i} \frac{i!}{\alpha! \beta!} t^\alpha \otimes t^\beta$$

$$= \sum_{\alpha+\beta=i} \gamma_\alpha \otimes \gamma_\beta$$

$$\therefore \text{Tor}_{A_R}(R, R) = \Gamma(t) \otimes E(w)$$

as Hopf algebras, if  $R = \mathbb{Z}$ .

But  $\mathbb{Z}$  is the universal case via:

$$\text{Tor}_{A_R}(R, R) = \text{Tor}_{A_R}(\mathbb{Z}, \mathbb{Z}) \otimes R.$$