Lecture 4. Hopf invariants and Bockstein spectral sequences

Let p be an odd prime and localize at p throughout this lecture:

Goal: To show $\begin{cases} p\, H_*(\Omega^2 S^{2n+1}; \mathbb{Z}_{(p)}) = 0, \\ * \neq 0, 2n-1 \end{cases}$

Translation to the mod p homology Bockstein spectral sequence:

$$E^1 = H(\Omega^2 S^{2n+1}; \mathbb{Z}/p) = E(\tau_0, \tau_1, \dots) \otimes P(\sigma_1, \sigma_2, \dots)$$

with degree $\tau_i = 2p^i n - 1$, degree $\sigma_i = 2p^i n - 2$.

Claim: $\beta^1 \tau_i = \sigma_i \quad \forall i \geq 1$

$\therefore\ E^2 = E(\tau_0)$

$\therefore\ E^2 = E^3 = \dots = E^\infty$

Step 1. Check the lowest dimensional case

$$H_*(\Omega^2 S^{2n+1}; \mathbb{Z}_{(p)}) = \mathrm{Cotor}^{P[x]}(\mathbb{Z}_{(p)}, \mathbb{Z}_{(p)})$$

$P[x] = H_*(\Omega S^{2n+1}; \mathbb{Z}_{(p)})$ or ;