Step 2 Recall the Hopf maps localized at p

Theorem: $\exists$ maps $h : \Omega S^{2n+1} \longrightarrow \Omega S^{2pn+1}$ such that $h : H(\Omega S^{2n+1}) \to H(\Omega S^{2pn+1})$ is an epimorphism of coalgebras.

Corollary: The linear retraction $r : H(\Omega S^{2n+1}) \longrightarrow \langle 1, \iota, \ldots, \iota^{p-1} \rangle$ defines an isom Θ of right comodules

$$\begin{tikzcd}
H(\Omega S^{2n+1}) \arrow[r, "\Delta"] \arrow[dr, "\Theta"'] & H(\Omega S^{2n+1}) \otimes H(\Omega S^{2n+1}) \arrow[d, "r \otimes h"] \\
 & \langle 1, \ldots, \iota^{p-1} \rangle \otimes H(\Omega S^{2pn+1})
\end{tikzcd}$$

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mod p, Θ is an isom of coalgebras and can be iterated to give the decomposition of the previous lecture

$$H(\Omega S^{2n+1}) = \bigotimes_{k=0}^{\infty} \langle 1, \iota^{p^k}, \ldots, \iota^{(p-1)p^k} \rangle$$

and leads to