is equivalent to its adjoint

$$\begin{tikzcd} \Sigma X \arrow[r] & Y \\ \Sigma \Omega S^{2n+1} \arrow[u, leftarrow, "\Sigma i"'] \arrow[ur, dashed] & \end{tikzcd}$$

But the James decomp $\Longrightarrow$ that Σi has a retraction. //

A quick proof of the James decomp!

Multiply the maps $\Sigma : S^{2n} \to \Omega S^{2n+1}$ by itself k-times to get

$$\begin{tikzcd} \mu_k : S^{2n} \times \cdots \times S^{2n} \arrow[r, "\Sigma \times \cdots \times \Sigma"] \arrow[dr] & \Omega S^{2n+1} \times \cdots \times \Omega S^{2n+1} \arrow[d, "\text{mult}"] \\ & \Omega S^{2n+1} \end{tikzcd}$$

Use the decomposition

$\Sigma (X \wedge Y) \vee \Sigma X \vee \Sigma Y \simeq \Sigma (X \times Y)$ to get

$$\begin{tikzcd} \eta_k : S^{2nk+1} = \Sigma (S^{2n} \wedge \cdots \wedge S^{2n}) \arrow[r] \arrow[dr] & \Sigma (S^{2n} \times \cdots \times S^{2n}) \arrow[d, "\Sigma \mu_k"] \\ & \Sigma \Omega S^{2n+1} \end{tikzcd}$$