Proposition. Use $\mathbb{Z}_{(p)}$ coefficients and let $\alpha : \Gamma(y) \to \Gamma(x)$ be a map of algebras such that $\alpha(y) = \gamma_p(x)$. Then

1) $\Gamma(y)$ is mapped isom onto the subalgebra A_p of $\Gamma(x)$ which is concentrated in degrees divisible by $2pn$.

2) $\Gamma(x)$ is a free $\Gamma(y)$ module with basis $1, \gamma_1(x), \ldots, \gamma_{p-1}(x)$.

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1) we need to check

$$\alpha[\gamma_i(y)] = u_i \, \gamma_{pi}(x)$$

where u_i is a unit in $\mathbb{Z}_{(p)}$.

$$\alpha[\gamma_i(y)] = \alpha\left(\frac{y^i}{i!}\right) = \frac{\gamma_p(x)^i}{i!} = \frac{\left(\frac{x^p}{p!}\right)^i}{i!}$$

$$= \frac{x^{pi}}{(p!)^i \, i!} = \frac{(pi)!}{(p!)^i \, i!} \, \gamma_{pi}(x)$$

$= u_i \, \gamma_{pi}(x)$ and the formula for the number of primes in a factorial $\Longrightarrow \nu_p(u_i) = 1$