

2) we need to check that:

$$1 \leq k \leq p-1 \text{ and } j \geq 1 \Rightarrow$$

$$\gamma_k(x) \gamma_{p^j}(x) = \binom{p^{j+k}}{k} \gamma_{k+p^j}(x)$$

where $\binom{p^{j+k}}{k}$ is a unit in $\mathbb{Z}/(p)$.

$$\text{But } \binom{p^{j+k}}{k} \equiv \binom{p^j}{0} \binom{k}{k} = 1 \cdot 1 = 1 \pmod{p} //$$

Closing exercises: 1) Localized at a prime p ,

$$\mathbb{S}^{2n} \vee \mathbb{S}^{4n} \vee \dots \vee \mathbb{S}^{2n(p-1)} \hookrightarrow \mathbb{S}^{2n+1} \xrightarrow{h} \mathbb{S}^{2n+1}$$

is a fibration sequence up to

homotopy equivalence.

2) Show that the Hopf algebra $H(\mathbb{R}^2 \mathbb{S}^{2n+1}, \mathbb{Z}/(p))$ is primitively generated for $p = \text{odd prime}$.

Hint: Use the following theorem of Milnor-Moore:

Let B be a connected Hopf algebra over $\mathbb{Z}/(p)$ and suppose B has commutative multiplication. If

$\mathcal{G}(B) =$ the sub Hopf algebra generated by all p -th powers, there is an exact sequence

$$0 \rightarrow \mathcal{P}(\mathcal{G}(B)) \rightarrow \mathcal{P}(B) \rightarrow \mathcal{Q}(B).$$