Arolla 2012:
Abstracts of talks

Ilias Amrani: *Cofibrantly generated model structure on infinity groupoids*

We will define a notion of topological infinity groupoid associated to a topological category. It is well known that the category $\infty - Grp$ of infinity groupoids is a model category. In this talk, we will prove that $\infty - Grp$ is actually a cofibrantly generated model category, Quillen equivalent to the category of simplicial sets. As an application we construct a functorial Dwyer-Kan localization for topological categories. We will compare the notion of coherent nerve and the standard nerve (by forgetting the topology) for some particular infinity groupoids using the derived internal hom, initialy defined by B. Toën for dg-categories.

Dave Barnes: *Orthogonal Calculus and Model Categories*

(Joint work with P. Oman)

Orthogonal calculus is a calculus of functors, inspired by Goodwillie calculus. It takes as input a functor from finite dimensional inner product spaces to topological spaces and as output gives a tower of approximations by well-behaved functors. The output captures a lot of important homotopical information and is an important tool for calculations.

In this talk I will report on work in which we use model categories to improve the foundations of orthogonal calculus. This provides a cleaner set of results and makes the role of $O(n)$-equivariance clearer. The classification of n-homogeneous functors in terms of spectra with $O(n)$-action can then be phrased as a zig-zag of Quillen equivalences. As an application, we develop a stable variant of orthogonal calculus with topological spaces replaced by orthogonal spectra.

Arthur Bartels: *The 3-category of conformal nets*

(Joint work with C. Douglas and A. Henriques)

I will discuss the construction of a 3-category whose objects are conformal nets. This 3-category deloops the 2-category of von-Neumann algebras, bimodules and intertwinners.

Clemens Berger: *Goodwillie calculus for Gamma spaces*

(Joint work with G. Biedermann)

In the early 70's, Gamma-spaces have been introduced by Segal as a combinatorial way of representing connective spectra. Later on, Bousfield and Friedlander equipped Gamma-spaces with two distinct Quillen model structures: a strict and a stable one; the first arises from considering Gamma-spaces as certain endofunctors of the category of based topological spaces, while the second derives from the first by localization with respect to stable equivalences.

The purpose of this talk is to define, for each Gamma-space A, a tower of strict fibrations $P_{n+1}A \to P_nA$ such that the associated tower of endofunctors coincides with the Goodwillie tower of the endofunctor of A. In other words, we present a way of restricting "Goodwillie-calculus" to the category of Gamma-spaces. This recovers of course Segal's characterization of connective spectra as "very special" Gamma-spaces, but it also recovers results of Mauer-Oats stating roughly that, inside the category of Gamma-spaces, Goodwillie's n-excisive functors may be characterized by the vanishing of the $(n+1)$-st cross effects.

We finally show how the calculation by Rognes and Johnson of the Goodwillie derivatives of the identity functor may be carried out (entirely combinatorially) inside the category of Gamma-spaces.

Gunnar Carlsson: *Representations and K-theory of fields*

We will discuss some results which construct the K-theory spectrum of a field as a derived completion construction on the spectrum associated to the symmetric monoidal category of continuous representations of its absolute Galois group. It will in particular give an identification of homotopy groups of derived completions of the representation ring of the Galois group with completed Milnor K-theory.

Wojciech Chachólski: *Idempotent deformations and dynamics of finite groups*

(Joint work with M. Blomgren, E. Dror Farjoun, and Y. Segev)

The proofs of Ravenel's conjectures and their reinterpretations in the form of the classification of the homotopy idempotent functors of spectra that commute with telescopes by Devintaz-Hopkins-Smith were a culmination of a few decades of progress achieved in stable homotopy theory. The simplicity of this classification is remarkable. The category of restrictions of these functors to p-local finite spectra is isomorphic to the poset of natural numbers. This was generalized by Bousfield to also a remarkably simple classification of so called Bousfield localizations of finite p-local spaces. After these classifications, there was a hope (even published wrong results) that a similarly explicit classification might be true for idempotent functors of topological spaces that commute with telescopes. This however turned out to be a failed hope since the category of idempotent functors of topological spaces surjects onto the lattice of all ideals in the ring of stable homotopy groups of the sphere. To understand possible difficulties one strategy has been to ask analogous classification questions in other settings, for example derived categories of rings or groups, with a hope that the results might shed some light on possible difficulties in the case of topological spaces.

I believe however that the simplicity of the stable analog is misleading and in general one should not expect to be able to classify such functors. Instead we should focus on their action. I believe that its is not the functors globally, but the orbits of their action that often have an explicit and remarkably simple classifications. The aim of the talk is to describe this action in the category of groups. Although it is not reasonable to expect any classification of idempotent functors of groups, I will give an explicit classification of the orbits of their action on finite groups. I will show for example that except projective linear groups, the orbits of simple groups can have at most 7 elements.

Michael Ching: *Calculus of homotopy functors and modules over the little disc operads*

(Joint work with G. Arone)

The goal of this talk is to describe how the Goodwillie tower of a functor from based spaces to spectra is built from its homogeneous layers. I will describe various structure maps that connect the coefficient spectra of these layers and relate this description to right modules over the little disc operads. This approach gives us a formula for the terms in the Goodwillie tower as mapping spectra for coalgebras over certain comonads. I will also discuss how our work applies to certain interesting functors, including stable mapping spaces for manifolds and Waldhausen's algebraic K-theory of spaces.

Dan Dugger: *Motivic stable homotopy groups of spheres*

(Joint work with D. Isaksen)

I will give a survey of various things we know—or don't know—about the groups in the title. I will also give some indication of why we care.

Benoît Fresse: *Rational homotopy of operads and little discs*

The first purpose of this talk is to explain the general definition of a Sullivan model for the rational homotopy of topological operads. The usual Sullivan algebra of piecewise linear forms, classically used to define the model of a space, is not strictly monoidal, and can not be directly applied to get a rigid model for operads. I will explain how to work out this problem precisely.

Then I will address applications of the rational model for the operad of little 2-discs. To be specific, I will explain that the formality of the chain operad of little 2-discs can be upgraded to a formality statement for the rational operadic model, and I will give a topological interpretation of this result.

In Tamarkin's approach, a formality quasi-isomorphism for little 2-discs is associated to any Drinfeld's associators. My main result asserts that we actually have a one-to-one correspondence between the Drinfeld associators and the homotopy classes of formality quasi-isomorphisms for the rational operadic model of little 2-discs.

Finally, I will briefly discuss some qualitative aspects of this result and their possible (conjectural) generalizations.

Teena Gerhardt: *Algebraic K-theory and Witt vectors*

(Joint work with V. Angeltveit, M. Hill, and A. Lindenstrauss)

Algebraic K-theory brings together classical invariants of rings with difficult computations in homotopy theory. The connection is through trace maps relating algebraic K-theory to fixed point spectra of topological Hochschild homology (THH). The fixed point spectra of THH are closely related to Witt vectors, and this relationship can facilitate K-theory computations. In this talk I will discuss connections between algebraic K-theory and Witt vectors. In particular I will describe new algebraic K-theory computations which naturally lead to an n-dimensional generalization of the big Witt vectors.

Brayton Gray: *Some new techniques in unstable homotopy theory*

(Joint work with S. Theriault)
The Anick spaces play a key role in the secondary suspension providing secondary EHP sequences. A recent reconstruction gives a greatly simplified construction of these spaces for all odd primes. This allows an attack on some of the conjectures implied by the EHP sequences. I will indicate the ingredients in the proof that the Anick spaces are homotopy commutative and homotopy associative H-spaces at primes bigger than 3 and have an interesting universal property reminiscent of some old results of Barratt on the growth of torsion.

Michael Joachim: *Classification of twists in equivariant K-theory for proper and discrete actions*

(Joint work with Bárcenas, Espinoza and Uribe)
We define the equivariant K-theory twisted by a so-called projective unitary stable bundle and construct a universal projective unitary stable bundle for proper actions of discrete groups. We will show how to calculate the homotopy type of the classifying space for projective unitary stable and equivariant bundles and that in the case of a finite group action, the isomorphism classes of projective unitary stable and equivariant bundles are classified by the third equivariant integral cohomology group. Our results extend and generalize results of Atiyah-Segal.

Anssi Lahtinen: *String topology of classifying spaces*

(Joint work with R. Hepworth)
Analogous to string topology of manifolds, the string topology of classifying spaces studies the homology of the free loop space of a compact Lie group G. In the original work on the subject, Chataur and Menichi showed that the homology of the free loop space of BG is the value of a circle in a closed non-unital non-counital Homological Conformal Field Theory, a field theory where operations are parametrized by homology classes of classifying spaces of diffeomorphism groups of surfaces. In this talk, I will discuss a novel construction of the string topology of BG. The new construction shifts emphasis from diffeomorphisms to homotopy equivalences, and radically enlarges the class of allowable cobordisms, trading surfaces with boundary for arbitrary spaces homotopy equivalent to finite 1-dimensional CW-complexes. What results is a new kind of field theory related to both the mapping class groups of surfaces and automorphism groups of free groups with boundaries, and containing within it a counital open-closed HCFT analogous to the full HCFT constructed by Godin in the manifold case.

Ian Leary: *A metric Kan-Thurston theorem*

A locally CAT(0) metric is a generalization of a Riemannian metric of non-positive curvature; in particular any space admitting such a metric is aspherical. Gromov gave an easy combinatorial criterion for a cubical complex to admit such a metric. CAT(0) cubical complexes have played a vital role in recent major developments in 3-manifold theory, and a role in some aspects of geometric topology. I shall present an application to algebraic topology: proofs of the Kan-Thurston theorem and other related results using CAT(0)-cubical complexes.

Kathryn Lesh: *Bredon homology of partition complexes*

(Joint work with G. Arone and W. Dwyer)
I will discuss work on the computation of the homology groups of partition complexes with certain Bredon coefficient systems. It turns out that if n is not a power of a prime, then these are trivial in a strong sense, while if n is a prime power, one gets a description involving a Tits building. The calculation uses techniques of homology approximations due to Dwyer, applied in the context of more general coefficient systems.

Ran Levi: *The algebraic structure of finite loop spaces*

(Joint work with C. Broto and B. Oliver)
Finite loop spaces were and remain among the most fundamentally important objects of study in homotopy theory. We apply the theory of p-local compact groups to study of a family of spaces, of which p-completed classifying spaces of finite loop spaces are an example.

The concept of p-local compact groups generalizing that of p-local finite groups, gives a solid and very general algebraic framework in which the p-local homotopy theory of classifying spaces satisfying certain finiteness properties can be studied. Within this setup, many general properties, common to the entire family can be established, and research in this direction is still very much on the go. Examples of p-local compact groups arise in particular from the classifying spaces of compact Lie groups, p-compact groups, and linear torsion groups. There are also a number of exotic examples which are not known to belong to any of the above.

In this talk we discuss a very general theorem which states that if $X \to E$ is a regular finite covering space, with X the classifying space of a p-local compact group, then up to p-completion, so is E. As a consequence it follows that if E is any space whose loop space has finite mod p cohomology, then the p-completion of E is the classifying space of a p-local compact group.

Ib Madsen: *Automorphism groups of some highly connected manifolds*

I will use surgery techniques and rational homotopy theory to describe homological stability results for block diffeomorphisms of some $(n-1)$-connected $2n$-dimensional smooth manifolds. In the concordance stable range this gives cohomological information about the diffeomorphism groups of the high dimensional analogue of oriented surfaces upon using recent results of Galatius and Randal-Williams.

Angélica Osorno: *Towards modeling homotopy n-types of spectra*

(Joint work with N. Johnson)

It is a classical result that groupoids model homotopy 1-types, in the sense that there is an equivalence between the homotopy categories, via the classifying space and fundamental groupoid functors. We extend this result to stable homotopy 1-types and Picard groupoids, that is, symmetric monoidal groupoids in which every object has a weak inverse. Using an algebraic description of Picard groupoids, we identify the Postnikov data associated to a stable 1-type; the abelian goups π_0 and π_1, and the unique k-invariant.

We also briefly explore the case for $n = 2$, where we expect stable homotopy 2-types to be modelled by Picard bigroupoids.

Irakli Patchkoria: *Rigidity in equivariant stable homotopy theory*

For any finite abelian 2-group G, we show that the 2-local G-equivariant stable homotopy category, indexed on a complete G-universe, has a unique equivariant model in the sense of Quillen model categories. This means that the suspension functor, homotopy cofiber sequences and the stable Burnside category determine all "higher order structure" of the 2-local G-equivariant stable homotopy category such as for example equivariant homotopy types of function G-spaces. The theorem can be seen as an equivariant generalization of Schwede's rigidity theorem at prime 2.

Kári Ragnarsson: *A homotopy characterization of p-completed classifying spaces of finite groups*

(Joint work with M. Gelvin)

The homotopy type of classifying spaces of finite groups is easy to characterize: they are the Eilenberg-MacLane spaces of type $K(G,1)$. However, the p-local case is more complicated, as Oliver's proof of the Martino–Priddy conjecture shows that the algebraic model corresponding to p-completed classifying space of a finite group is its p-local fusion system of conjugation among

p-subgroups. This suggests that, to model p-completed classifying spaces, one should generalize to include Puig's notion of abstract fusion systems. To this end, Broto–Levi–Oliver introduced p-local finite groups, a somewhat complicated algebro-categorical model that is widely agreed to capture the essential homotopical properties of p-completed classifying spaces. Around the same time, Haynes Miller proposed a purely homotopy-theoretic model consisting of a space that admits a transfer retract to the classifying space of a finite p-group, and tentatively suggested that they should be equivalent to the Broto–Levi–Oliver model. In this talk I will explain these two models and then outline a proof of Miller's conjecture, assuming unpublished work of Lannes on the Segal conjecture.

Oscar Randal-Williams: *Stable moduli spaces of high dimensional manifolds*

(Joint work with S. Galatius)

I will discuss recent work, in which we generalise the Madsen–Weiss theorem from the case of surfaces to the case of manifolds of higher even dimension (except 4). In the simplest case, we study the topological group $\mathcal{D}_g$ of diffeomorphisms of the manifold $\#^g S^n \times S^n$ which fix a disc. We have two main results: firstly, a homology stability theorem—analogous to Harer's stability theorem for the homology of mapping class groups—which says that the homology groups $H_i(B\mathcal{D}_g)$ are independent of g for $2i \leq g-4$. Secondly, an identification of the stable homology $H_*(B\mathcal{D}_\infty)$ with the homology of a certain explictly described infinite loop space—analogous to the Madsen–Weiss theorem. Together, these give an explicit calculation of the ring $H^*(B\mathcal{D}_g; \mathbb{Q})$ in the stable range, as a polynomial algebra on certain explicitly described generators.

Oriol Raventos: *Adams representability for well generated triangulated categories*

(Joint work with F. Muro)

A well generated triangulated category T is said to satisfy α-Adams representability if every contravariant functor $T^\alpha \longrightarrow Ab$ that sends coproducts of less than α objects to products and triangles to long exact sequences (where T^α is the full subcategory of α-compact objects for a fixed regular cardinal α) is isomorphic to the restriction of a representable functor $Hom(-, X)|_{T^\alpha}$.

If $\alpha = \aleph_0$, the stable model category salifies Adams representability (as proved by Adams, inferring the representability of homology theories). This was later generalized by Neeman to a broader family of triangulated categories.

We will give conditions to a triangulated category in order to satisfy α-Adams representability with respect to an arbitrary regular cardinal. We will provide many concrete positive examples for $\alpha = \aleph_1$ as well as examples that do not satisfy α-Adams representability for any infinite cardinal α..

Birgit Richter: *A spectral sequence for the homology of a finite algebraic delooping*

In the world of chain complexes E_n-algebras are the analogues of n-fold loop spaces. Fresse showed that operadic E_n-homology of an E_n-algebra computes the homology of an n-fold algebraic delooping. In joint work with Stephanie Ziegenhagen we develop spectral sequences, a resolution and a composite functor spectral sequence, that help to compute these deloopings. I'll talk about the set-up and about some concrete examples.

Constanze Roitzheim: *Simplicial, stable and local framings*

One key objective in stable homotopy theory is finding Quillen functors between model categories. Framings provide a way to construct and classify Quillen functors from simplicial sets to any given model category. There is also a more structured set-up where one studies Quillen functors from spectra to a stable model category. We will investigate how this is compatible with Bousfield localisations and how it can be used to study the deeper structure of the stable homotopy category. For example, among other applications, framings allow us to prove that the E-local stable homotopy category does not possess an algebraic model unless E is rational homology.

Steffen Sagave: *Logarithmic structures on K-theory spectra*

In order to explain various phenomena arising in connection with the topological Hochschild homology and algebraic K-theory of structured ring spectra, John Rognes has introduced a notion of ring spectra with logarithmic structures. We will explain how a slight modification of his approach enables us to define interesting logarithmic structures on connective K-theory spectra. These give rise to intermediate objects between connective and the periodic K-theory spectra. Moreover, these logarithmic structures exhibit the inclusion of the p-complete Adams summand into the p-complete complex connective K-theory spectrum as a formally log étale map, that is, a map whose logarithmic topological André-Quillen homology vanishes. By an analogy with local number rings, this confirms that the inclusion of the Adams summand should be viewed as a tamely ramified extension of structured ring spectra.

Gonçalo Tabuada: *The fundamental isomorphism conjecture via noncommutative motives*

(Joint work with Paul Balmer)

Making use of the theory of noncommutative motives we introduce the fundamental isomorphism conjecture and describe it in terms of algebraic K-theory.

Andy Tonks: *Unital associahedra*

(Joint work with F. Muro)

In this talk I present work (arxiv:1110.1959, to appear Forum Math) on the construction of the unital associahedra. These form a cellular topological operad uAss_∞, resolving the operad governing topological monoids, and with cellular chains precisely the dg operad of Fukaya-Ono-Oh-Ohta. They are not polytopes: in fact they are not even finite dimensional.

The classical associahedra are polytopes introduced by Stasheff to parametrize the multivariate operations naturally occurring on loop spaces of connected spaces. These form a topological operad Ass_∞, resolving the operad Ass governing spaces-with-associative-multiplication, and with cellular chains the dg operad governing A_∞-algebras (that is, a resolution of the operad governing associative algebras).

In classical applications it was not necessary to consider units for multiplication, or it was assumed units were strict. The introduction of non-strict units into the picture was considerably harder: Fukaya–Ono–Oh–Ohta introduced homotopy units for A_∞-algebras in their work on Lagrangian intersection Floer theory, and equivalent descriptions of the dg operad for homotopy unital A_∞-algebras have now been given, for example, by Lyubashenko and by Milles–Hirsch.

Antoine Touzé: *From representation theory of GL_n to algebraic topology computations*

Strict polynomial functors were invented by Friedlander and Suslin, in connection with representation theory of GL_n. They used these functors to perform calculations to prove the longstanding conjecture that finite group schemes have finite type cohomology algebras.

In this talk I will explain how some calculations with strict polynomial functors are related to classical algebraic topology, and give applications of this link to algebraic topology and to representation theory of GL_n.