AROLLA CONFERENCE ON ALGEBRAIC TOPOLOGY:

ABSTRACTS

Greg Arone, University of Virginia:

A curious construction on K-theory

We discuss a general construction that takes a Gamma category à la Segal as an input, and produces a certain filtration of spectra as an output. When the input is the category of finite sets, our construction gives the filtration of the Eilenberg- Maclane spectrum by symmetric powers of the sphere spectrum. When the input is the category of complex vector spaces, we get an apparently new filtration of complex K-theory. This filtration has formal properties that are suprisingly similar to the properties of the symmetric power filtration. In particular, we conjecture that an analogue of Kuhn's theorem on the symmetric power filtration (what used to be known as the "Whitehead conjecture") holds. The conjecture does not have an analogue for real K-theory. This is joint work with Kathryn Lesh

Christian Ausoni, Universität Bonn:

On the algebraic K-theory of complex K-theory

Let $K(ku)$ be the algebraic K-theory spectrum of the connective complex K-theory spectrum ku, completed at a prime $p > 3$. We will give motivation for our interest in this spectrum, and present a computation of its mod p and mod v_1 homotopy groups.

Clemens Berger, Université de Nice:

The Barratt-Eccles operad and its canonical filtration

The cotriple resolution of the permutation operad defines a simplicial E_∞-operad which has been studied by Barratt and Eccles in the early 70's in order to recognize infinite loop space structures. Applying the normalized chain functor one gets an E_∞-chain operad $\mathcal{E}$ whose remarkable properties have been discovered only recently. For instance, the singular cochains of any topological space carry a natural $\mathcal{E}$-action. On a more algebraic side, the Hochschild cochains of any unitary associative algebra carry an action by an E_2-suboperad of $\mathcal{E}$ (this solves the so called Deligne conjecture).

In this expository talk, I will summarize my joint work with Benoît Fresse on this operad and trace connections to related work by Jim McClure and Jeff Smith.

Natalia Castellana, Universitat Autonoma de Barcelona:

H-spaces with finitely generated cohomology as algebras over the Steenrod algebra

We characterize Hopf spaces with finitely generated cohomology as algebras over the Steenrod algebra. The main structure result is the following: let X be a connected H-space with finitely generated cohomology as algebra over the Steenrod algebra. Then X is the total space of an H-fibration whose base space has finite mod p cohomology and the fiber is a finite-ype p-torsion Postnikov piece. The techniques used in the proof include Bousfield's nullification functor and Lannes' T-functor to describe topologically the H-spaces whose module of indecomposables $QH^*(X; F_p)$ lies in some stage of the Krull filtration. We use this result to extend Hubbuck's torus theorem to describe homotopy commutative H-spaces in this setting. We also give a criterion on how to recognize connected covers of H-spaces with finite mod p cohomology.

Boris Chorny, University of Western Ontario

Homotopy localizations vs. ordinary localizations

A theorem of Adamek, Rosicky and Trnkov (1987) asserts, in particular, that, under weak Vopenka principle, every localization functor in a locally presentable category is a localization with respect to a set of maps. Under full Vopenka principle, we were able to show (with Carles Casacuberta), that every simplicial homotopy localization functor in a combinatorial simplicial model category is a localization with respect to a set of maps. We will argue in this talk that a combinatorial model category is the correct generalization of the notion of a locally presentable category. That means the assumption on the model category to be cofibrantly generated is essential. In more detail, we will present an example, which arises from the equivariant homotopy theory, of a locally presentable, non-cofibrantly generated model category and a localization functor which is not a localization with respect to any set of maps.

Brayton Gray, University of Illinois at Chicago:

Decomposition theory

We describe a Krull-Schmidt theorem for decomposing H-spaces and simply connected co-H-spaces, and use this to construct a 1-1 correspondence between atomic co-H- spaces G and atomic H- spaces T. This correspondence yields a decomposition of the loop space of G as a product of T and the loops on a co-H- space R constructed out of G. This, and a "Hopf fibration" relating these spaces, generalizes the decomposition of Cohen- Neisendorfer-Moore and Anick-Gray, as well as many other examples. One of the applications of this involves analyzing the higher smash products of two-cell complexes and relies on the representation theory of the symmetric group

Jesper Grodal, University of Chicago:

Root systems for 2-compact groups

p-compact groups are homotopy versions of compact Lie groups, but with all the structure concentrated at a single prime p. The classification of p-compact groups, p odd, as proved by Andersen-myself-Moller-Viruel, states that there is a 1-1 correspondence between connected p-compact groups and finite reflection groups over the p-adic integers. For $p = 2$ the conjectural statement is a modification of this statement replacing reflection groups by "root data". So, as a first step towards a classification for $p = 2$, one has to come to terms with the concept of the root datum of a 2-compact group, and understand how it relates to the maximal torus normalizer. In my talk I will explain this step, which is a combination of work by Dwyer-Wilkerson and Andersen and myself.

Lars Hesselholt, MIT

On bi-relative algebraic K-theory

This is joint work with T. Geisser. It was recently proved by G. Cortiñas that, rationally, bi-relative algebraic K-theory and bi-relative cyclic homology agree. We show that, with p-adic coefficients, bi-relative algebraic K-theory and bi-relative topological cyclic homology agree. This may be used to evaluate the algebraic K-theory of singular schemes. For instance, it follows that for *any* curve over a perfect field k of characteristic $p > 0$, the cyclotomic trace induces an isomorphism of the p-adic algebraic K-theory and the p-adic topological cyclic in non-negative degrees. For the nodal curve we find that the p-adic algebraic K-theory is concentrated in even degrees and that the group in degree $2n$ is canonically isomorphic to the underlying additive group of the ring $\mathbf{W}_n(k)$ of big Witt vectors of length n in k.

Ran Levi, Aberdeen:

The homotopy theory of fusion systems

A saturated fusion system over a finite p-group S is a category whose objects are the subgroups of S, and whose morphisms satisfy certain axioms. In earlier work Broto, Levi and Oliver defined and studied "centric linking systems" associated to saturated fusion systems. These are categories with enough extra structure to allow associating a "classifying space" with the given fusion system. This enabled them to define p-local finite groups, which create a firm link between the purely algebraic concept of a fusion system and homotopy theory. In this talk we shall study a number of naturally arising questions related to p-local finite groups. For instance we will investigate the question of what constitutes a morphism between two p-local finite groups, and indicate the difficulties in defining p-local finite groups as a category. We will also consider "extensions" of p-local finite groups, and draw the partial analogy with extensions of ordinary groups. Time permiting, we will report on some other recent developments in the subject and new lines of investigation.

Jim Lin, University of California–San Diego:

Finite H-spaces that are not loop spaces

In recent years, there has been much progress on the classification of p- complete finite loop spaces also known as mod p compact groups. It has been shown that they possess Weyl groups and maximal tori. Further they decompose into simple factors whose cohomology over the Steenrod algebra is completely known. However, if a finite H-space does not have a loop space structure, there is no analogous classification. In this talk, we will consider finite H-spaces that are not loop spaces and give a survey of known results and open problems.

Mike Mandell, University of Chicago

E_1, E_2, E_3, E_4

An E_1 ring spectrum is what is usually called an A-infinity ring spectrum or just "S-algebra". An E_1 ring spectrum has a nice stable category of modules. An E_2 structure gives this category a monoidal product. An E_3 structure gives this monoidal category a braided monoidal structure. If the E_3 structure extends to an E_4 structure, the braided monoidal structure is a symmetric monoidal structure. The construction suggests a category-theoretical generalization of the Deligne conjecture to which it provides a partial converse.

Guido Mislin, ETHZ:

Traces in K-theory of group C^*-algebras and the Bass conjecture

The classical Bass Conjecture predicts that the Hattori-Stallings trace on the K-theory of the complex group algebra $\mathbb{C}G$ of a groups G maps in to the subspace spanned by conjugacy classes of elements of finite order. An ℓ^1-version of that conjecture has been considered recently (cf. Berrick-Chatterji-M: Math. Ann. 2004). Here we will study a natural analog for some other Banach algebra completions of $\mathbb{C}G$. We prove in particular that if G satisfies the Baum-Connes conjecture, then for the reduced group C^*-algebra C_r^*G, the universal (Hattori-Stallings) trace $K_0C_r^*G \to HH_0^{top}C_r^*G$ takes values in the subspace spanned by the images of the torsion elements in G; applications to the center-valued trace are presented.

Ieke Moerdijk, University of Utrecht:

Monoidal model categories and the Boardman-Vogt resolution of operads

This is a report on joint work with C. Berger. We show that the Boardman-Vogt resolution of operads can be constructed in any monoidal model category with a suitable interval, and provides explicit cofibrant resolutions in this context. We investigate the functorial properties of this resolution, and compare it to other known resolutions, notably the bar-cobar resolution (Hinich and others) and the Godement resolution.

Holger Reich, University of Oslo:

Algebraic K-theory of group rings and topological cyclic homology

The talk reports on joint work with J. Rognes, W. Lück and M. Varisco. We use topological cyclic homology and the cyclotomic trace to detect elements in the rationalized higher algebraic K-theory of integral group rings. Modulo a conjecture in number theory (known as the Schneider Conjecture – a higher analogue of the Leopoldt Conjecture), and under mild homological finiteness conditions on the group, we prove that the assembly map for connective algebraic K-theory and the family of virtually cyclic groups is rationally injective. This vastly generalizes a result of Bökstedt, Hsiang and Madsen and leads to a concrete description of a large direct summand inside the algebraic K-theory of an integral group ring. Along the way we also prove integral splitting and isomorphism results for THH- and TC-assembly maps.

Jérôme Scherer, CRM Barcelona:

A cellular dichotomy

(joint work with R. Flores)

We show that the $B\mathbb{Z}/p$-cellularization of the classifying space of a finite group G is either the classifying space of a finite group, or it has infinitely many homotopy groups. We study more precisely these homotopy groups in the second case.

Stefan Schwede, Universität Bonn:

Toda brackets and multiplications on 2-cell complexes

Let p be a prime. We explain the relationship between

(1) the level of coherence in the multiplication of a p-local 2-cell complex
(2) a certain homotopy operation in the p-local stable homotopy groups of spheres, and
(3) certain Toda brackets of length $2p-1$.

All this generalizes Toda's relation $\eta \cdot x \in \langle 2, x, 2\rangle$ for a stable homotopy element x satisfying $2x = 0$.

Brooke Shipley, University of Illinois at Chicago:

K-theory and derived equivalences

In joint work with Dan Dugger, we show that if two rings have equivalent derived categories then they have the same algebraic K-theory. In fact, if two rings are derived equivalent then the associated categories of complexes are Quillen equivalent. Thus, no homotopy invariant will distinguish between these two rings.

Stephen Theriault, Aberdeen:

Homotopy exponents of Lie groups of low rank

Fix a prime p. The homotopy exponent of a space X is the least power of p which annihilates the p-torsion in the homotopy groups of X. In this talk we will discuss upper bounds on the homotopy exponents of classical Lie groups of low rank. Cases of interest include: (i) $SU(3)$, $SU(4)$, and $Sp(2)$ at the prime 2, and (ii) $SU(n)$ at odd primes, for $n < p^2(p-1)$. These upper bounds are not known to be best possible, but are substantial improvements over previously known bounds and are close to the conjectured exponents. The approach is fairly straightforward and accessible. It makes use of the standard spherical resolutions and characteristic maps of the Lie groups, and known exponent information on spheres.

Nathalie Wahl, Aarhus University

Mapping class groups in dimensions 2 *and* 3 *and the automorphisms of free groups*

We prove that the map from the stable mapping class group of surfaces to the stable automorphism group of free groups induces an infinite loop map on the classifying spaces after plus construction. To show this, we introduce automorphisms of free groups with boundaries and use certain 3-dimensional manifolds to prove an appropriate homological stability theorem for these groups.

Sarah Whitehouse, University of Sheffield

Infinite sums of Adams operations

The talk will start with a review of results obtained in joint work with Francis Clarke and Martin Crossley, giving descriptions of various algebras of operations in p-local K-theory. Informally, one may say that the elements of these rings are a specified sort of infinite sum of Adams operations. We give explicit formulas for all the structure maps, involving the combinatorics of Gaussian polynomials. Results about the intricate ring structures are deduced.

I will then describe some joint work with Imma Gálvez, in which we explore the same kinds of sums of Adams operations for $MU_{(p)}$ and BP, thus defining the 'Adams subalgebra' of the rings of operations. We show that this is precisely the centre of the ring of degree zero operations.