

NEW TWISTS TO DUALITY IN ALGEBRA AND TOPOLOGY

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[Based on joint work with Benson, Dwyer and Iyengar.]

The notion of a Gorenstein local ring R is based on the homological algebra of the residue field k . This definition is readily adapted to the case of a ring spectrum R and a module spectrum k over it. For example we may consider $R = C^*(X; k)$, the ring spectrum of cochains on a space X . This is Gorenstein if X is a manifold or the classifying space of a finite group.

In commutative algebra, one is often more interested in the duality statements that follow from the Gorenstein condition, and this is true for ring spectra too. In the general context one obtains Poincaré duality for manifolds, Benson-Carlson duality for the cohomology of classifying spaces and Gross-Hopkins duality in chromatic homotopy theory as well as the familiar example. However, in general the duality will be twisted, and one requires an orientability condition to obtain duality in its simplest form. This is familiar for manifolds, but it is interesting for classifying spaces of compact Lie groups and it is the central feature of Gross-Hopkins duality.

Another useful fact in commutative algebra is that the Gorenstein condition localizes well, as is seen by local duality. This is of interest for classifying spaces of finite groups, where it can be used to prove a conjecture of Benson's.

The talk will discuss a selection of these topics.