The fixed points of THH: A generalization of the Segal conjecture.

Let B be a connective S-algebra. Then the cyclotomic trace map $K(B) \rightarrow TC(B)$ from the algebraic K-theory of B to its topological cyclic homology is almost an equivalence.

The topological cyclic homology of an S-algebra can be defined as an inverse limit where all the spectra involved are fixed point spectra of Bökstedt's topological Hochschild homology spectrum $THH(B)$.

The (strict) fixed points of a G-spectrum are in general very hard to calculate, and a standard way of approaching them is to compare with the homotopy fixed point construction. With the goal of calculating the homotopy groups of $TC(B)$, it will be important to study the relationship between the fixed points and the homotopy fixed points of THH with respect to cyclic p-subgroups of the circle group

$$\Gamma_C : THH(B)^C \rightarrow THH(B)^{hC}. \tag{1}$$

The homotopy fixed points on the right hand side is calculated by a homotopy fixed point spectral sequence. In favorable cases, the map Γ_C is an equivalence (after a suitable completion).

An important example where Γ_C is an equivalence after p-completion occurs when $B = S$ is the sphere spectrum. Then $THH(S) \simeq S$ and the question if the fixed points and the homotopy fixed points are the same is equivalent to the Segal conjecture (in one of its many formulations) for the group C.

In the case when C is the cyclic group of order two, the proof of the Segal conjecture was accomplished by W.H. Lin using calculations of certain Ext-groups.

In the talk we will see how to generalize the proof in this case to prove that (1) is a 2-adic equivalence when C is cyclic of order two and $B = BP$ is the Brown-Peterson spectrum.