

AN (ALLEGORICAL) INTRODUCTION TO HOMOTOPY CALCULUS OF FUNCTORS

BEN WALTER

The broad term “calculus of functors” can refer to one of three general types of constructions: *homotopy calculus of functors* (as constructed by Goodwillie), *orthogonal calculus of functors* (as constructed by Weiss), and *manifold calculus of functors* (as constructed by Goodwillie and Weiss). The goal of these calculi is to cleverly decompose a functor into simpler pieces in much the same way as the Taylor series decomposes a function in undergraduate calculus. These methods have been used in constructions and proofs in areas ranging from K -theory; Waldhausen’s A -theory; stable, unstable, and periodic homotopy theory; knot theory; the study of embeddings, immersions, and pseudoisotopies; and surgery theory.

This talk will be an *everyman* introduction to homotopy calculus of functors: I will give a very relaxed discussion in which the focus is on explaining intuition and giving very broad outlines of the main constructions and tools, rather than detailing proofs. Though the primary focus will be on homotopy calculus, the underlying metaphor driving the intuition and constructions will be framed so as to encompass homotopy calculus, orthogonal calculus, and (to some degree) manifold calculus. I will also comment on how one could attempt to construct similar theories of calculus in other frameworks and the primary difficulties of doing so. Time permitting, these comments will be underlined by the example of an incomplete construction of a “graph calculus of functors” which could be used to gain information on the chromatic numbers of graphs à la Kozlov and Lovasz.

DEPARTMENT OF MATHEMATICS, PURDUE UNIVERSITY, 150 N. UNIVERSITY STREET, WEST LAFAYETTE, IN 47907
E-mail address: `walterb@math.purdue.edu`