Performance of quantitative versus passive investing: a comparison in global markets

Robert C. Dalang, Christophe D. Osinski and Wolfgang Marty

Robert C. Dalang, Professor
Département de Mathématiques
Ecole Polytechnique Fédérale
1015 Lausanne
Switzerland
tél. (+41 – 21) 693 25 51
fax (+41 – 21) 693 43 03
e-mail : robert.dalang@epfl.ch

Christophe D. Osinski, Ingénieur mathématicien
Département de Mathématiques
Ecole Polytechnique Fédérale
1015 Lausanne
Switzerland
tél. (+41 – 21) 693 25 97
fax (+41 – 21) 693 42 50
e-mail : christophe.osinski@epfl.ch

Wolfgang Marty, Head of Portfolio Analytics
Crédit Suisse Asset Management
Giesshübelstrasse 30
P.O. Box 800
8070 Zurich
Switzerland
tél. (+41 – 1) 332 29 15
fax (+41 – 1) 333 88 55
e-mail : wolfgang.marty@csam.com

Performance of quantitative versus passive investing: a comparison in global markets

Robert C. Dalang, Christophe D. Osinski and Wolfgang Marty

Abstract

Every investor is interested in obtaining high returns with low risk. In this paper, we compare quantitative investment strategies based on the maximisation of expected utility with a strategy that seeks to minimise variance, and compare both of these quantitative strategies with passive index strategies. We describe an empirical study that shows that in global markets, the minimum variance strategy seems to consistently achieve lower risk and higher returns than a standard benchmark, while strategies that aim to maximise the risk adjusted return typically do exhibit higher returns but at the price of higher risk.

Passive and active investment strategies

The performance of an investment strategy is generally measured against a reference portfolio ('benchmark'). For example, the manager of a global equity fund usually chooses some global equities index as his or her particular benchmark.

A *passive* investor believes that it is improbable, over long periods of time, to consistently outperform the benchmark, and therefore, it is desirable to invest in a portfolio which simply holds securities in proportion to their weighting in the index. Passive investment strategies are attractive because of their low cost, and because the composition of the investor's portfolio is unaffected by market movements.

Active investors aim to outperform the benchmark. Active investors can be of at least two kinds: those that rely on knowledge and instinct, and those that use objective *quantitative* methods. For the latter, a mathematical model of the market is postulated, historical data from the markets are used to estimate the parameters of the model, and the investment strategy is computed mathematically within the model. Issues related to parameter estimation within the model and ease of computer implementation become important.

In principle, in order to achieve higher average returns, the investor must accept taking higher risk (volatility). Indeed, numerous studies of the U.S. equity market, which is highly efficient, confirm that a substantial number of portfolio managers are often unable to consistently outperform passive strategies (see for instance Philips, Rogers and Capaldi

[1996]). However, the study presented here shows that global equity markets are less efficient and therefore more promising for active investment. In particular, it appears that the *minimum variance strategy*, applied to global markets over a significant time period, is able to consistently achieve both lower risk and higher returns than passive strategies.

For the convenience of the reader, the main body of this article contains few mathematical formulas. For those who may want to reproduce our results or methods, these formulas are provided in the Appendix.

The mathematical model

We assume that the investor will limit his investments to portfolios that invest in the set of 36 FT sector indices listed in Exhibit 1. This provides the investor with ample diversification within the sectors. We do not consider here the possibility of stock selection within the different sectors.

The mathematical model assumes, in agreement with modern portfolio theory (see Merton [1992]) that the evolution of each FT sector index is statistically described by an equation whose main ingredients are the expected return and the volatility (see equation (1) in the Appendix). The return and volatility must be estimated on a month by month basis using historical data, as will be described further on. This leads to a non subjective (quantitative) approach to investing.

The issue faced by the investor is to decide, at each time t, which proportion of his capital should be invested in each sector. We assume that short selling is not allowed. For the passive investor, the choice is simple: this proportion is equal to the weight of this sector index in the global index. For the active quantitative investor, this proportion will be chosen in view of a particular goal to be achieved. We consider here various possible goals: (1) maximising the risk adjusted return, also called maximising the utility; (2) minimising the volatility, and (3) replicating the Index (passive strategy). The two quantitative strategies (1) and (2) will be described in the next two paragraphs.

Maximising the risk adjusted return

A natural objective for an investor is to seek to maximize the utility of his wealth c at some future date. We shall use the standard utility function

$$u(c) = \frac{1}{p} c^p,$$

where p is a non-zero real number such that $p < 1$. The positive quantity $1 - p$ is the Arrow-Pratt risk aversion index, which makes it possible to take into account the investor's risk tolerance: using a value of p close to 1 is suitable for an investor with high risk tolerance, while very negative values of p are preferable for a defensive investor. In our empirical studies, we shall consider an investor with high risk tolerance ($p = 0.5$) and another which is more defensive ($p = -10$).

A famous result of Merton [1971] (see also Merton [1992, chap. 5] and for instance Oksendal [1989, Example 11.5 p. 160]) tells us how to compute the investment strategy that maximizes the expected utility of the investor's future wealth: at each time t, the investor must assign weights to each sector index so as to maximize the expression

$$\text{portfolio return} - \frac{1-p}{2} \times (\text{portfolio volatility})^2.$$

In this study, no borrowing and no short selling are allowed, so the weight of each index must be non negative, and the sum of all weights, including the weight of the risk-free asset (the "bank account"), must be equal to 1. See (2) in the Appendix for the mathematical quadratic programming problem that makes it possible to compute the optimal weights on a computer, using standard software such as Matlab.

Minimising the volatility

The central premise of the well-known Markowitz theory is that higher returns can only be achieved at the price of higher risk. Rather than seek to maximize wealth, a risk-averse investor could seek to minimize risk. It is then interesting to see how much of a sacrifice on return this choice entails. We shall see that for global investors, the risk-return characteristics of such an approach can in fact be quite interesting.

We shall assume here that the portfolio is entirely invested in the risky assets (otherwise, if investment in the risk-free asset were allowed, zero volatiliy would be achieved by staying out of the market!). At each time t, the investor seeks to select the weight of each sector index so as to minimize the volatility of his portfolio. For the mathematical formulation, see (3) in the Appendix.

This objective leads to several significant differences compared with the one described above. As pointed out just above, the entire portfolio is invested in risky assets. In addition, expected returns do not influence the choice of the strategy, which only depends on the correlations between asset prices. From a computational point of view, this can be advantageous, because several studies have shown that errors in these estimates of returns have a significantly greater influence on quantitatively calculated portfolios than uncertainties regarding variances and covariances (see for instance Kalberg and Ziemba (1984) or Schäfer and Zimmermann (1998)). Furthermore, estimated variances and covariances based on historical time series do not seem to vary too greatly when the time series are updated, which should lead to portfolios with fewer transactions. Finally, the minimum variance strategy is not optimised against a benchmark portfolio, which can be convenient if there is no widely recognised such benchmark in the market under consideration.

Optimal strategies and the Markowitz framework

The theory of Markowitz states that the risk-return characteristics of all portfolios that are entirely invested in risky assets are contained in a "feasible region" delimited by a curve, called the efficient frontier (see Exhibit 2). The risk-return characteristics of the minimum variance portfolio are located at the point on the efficient frontier at which the tangent is vertical. It is well-known that it is never to the investor's advantage to select portfolios that *are not* on the efficient frontier.

The investor who seeks to maximise the risk adjusted return, and who can also invest in the risk-free asset, can construct portfolios with a wider range of possible risk-return characteristics. If short selling were allowed, then this investor's portfolio would be a linear combination of the risk-free portfolio and the *market portfolio,* with risk-return characteristics that lie on the *capital market line*, as shown in Exhibit 2.

For a *defensive* investor who seeks to maximise the risk adjusted return, the optimal portfolio will be partly invested in the risk-free asset: his portfolio lies on the capital market line, to the lower left of the market portfolio. As we assume that short selling is *not* allowed, the optimal portfolio for a *risk tolerant* investor will not lie on the capital market line, but on the efficient frontier, to the upper right of the market portfolio. The possible optimal portfolios considered in this study ($-\infty < p < 1$), are shown in bold in Exhibit 2.

The need for an empirical study

Minimum variance strategies at the stock level have already been analysed in detail (see for instance Kleeberg [1995]). Risk minimisation at the stock level, however, often gives rise to portfolios which invest in a limited number of securities only, and are thus less diversified. This can be remedied through the creation of additional restrictions with regard to individual positions, although this is not advisable since, in general, it increases portfolio risk.

Moreover, many analyses focus on minimum variance strategies in individual countries. This form of approach is not optimal because stock prices in a single country tend to be highly correlated, and minimum variance portfolios concentrated in a single country tend not to be very diversified. Furthermore, following the introduction of the Euro, an approach of this nature in Europe will not adequately reflect investor practices, since there is now no reason for a European investor to restrict himself to a single European country: in view of the globalisation of financial markets, many investors now operate in several countries, as do the companies they invest in.

While a mathematical analysis of strategies that maximise risk adjusted returns is now a part of the modern portfolio theory, it is still interesting to examine the behaviour of these strategies in the real market. Indeed, since global markets need not be as efficient as the theory assumes, the actual behaviour of these strategies may be different from what the theory suggests. Moreover, the strategies produced by a quantitative analysis will only be of interest if they outperform passive investment strategies, in terms of returns (even after transaction costs have been accounted for), lower volatility, or both.

Implementing optimal strategies

In order to make use of the mathematical models and strategies described above, it is necessary to estimate the parameters of the model, namely the future risk-free rate of return, the expected return, volatility and correlations of the FT indices, from observed monthly time series of the past risk-free interest rate and sector indices (a time step of one month seems to be reasonable for a long term investor). Then the optimal proportions are computed according to chosen objective and the portfolio is rebalanced so as to hold these proportions of each asset for one month. At the end of this month, a new estimation of the parameters is computed, which produces new proportions for the following month, etc. The parameters are estimated as follows.

Estimating the parameters

The value of each of the 36 FT sector indices is observed on the first day of each month, using data from January 1990 to July 1999.

Because the risk-free interest rate generally does not significantly change over a period of one month, the estimator of the risk-free interest rate for the next time period is simply the observed risk-free interest rate of the previous period.

For each sector index, we estimate its expected return during the next month using a 48-month moving average of observed (continuously compounded) returns (see formula (4) in the Appendix). Similarly, the volatility matrix is estimated from the covariance of the observed returns during the previous 48 months (see formula (5) in the Appendix).

Empirical results

Because the analyses are based on total return data for the FT Sector Indices provided by Goldman & Sachs for the period from January 1990 to July 1999, and because forty-eight months of past data are needed to estimate the various parameters according to the procedures described above, the quantitative strategies and portfolios are computed from February 1994 to July 1999. We consider four geographic regions: Global, USA, Europe and Japan, and we compare the performance of the risk adjusted returns strategy and the minimum variance strategy with the performance of the FT index for that region. We analysed several different portfolio characteristics:

1) *Performance of the quantitative strategies.* During the time period under consideration, Exhibit 3 shows that the minimum variance strategy and the risk-adjusted returns strategy outperform the FT Global Equities Index, a widely used benchmark for global markets. This also applies to the euro zone and Japan (Exhibit 4). In the USA, however, the minimum variance strategy underperforms the FT/US Index. This indicates that the US market largely fulfils theories of market efficiency, i.e. lower or even minimal risk is accompanied by lower return. Exhibit 5 shows that in phases of higher volatility of the FT Global Equities Index, the minimum variance portfolio can substantially and consistently reduce risk compared with this Index. Since the risk adjusted returns strategies are allowed to invest partly or entirely in the risk-free asset, they can exhibit lower

volatility over certain periods of time. This occurs in Exhibit 5 during 1994, for instance. However, the minimum variance strategy typically exhibits lower volatility than these strategies.

2) *Sharpe ratio*. One common way of assessing portfolio performance is the balance it achieves between risk and return. The Sharpe ratio $R(t)$ describes the relationship between risk and return for the portfolio less the return on risk-free investment. It is obtained by calculating the ratio between the volatility and the difference between the asset return and the risk-free rate of return (see formula (6) in the Appendix). A high Sharpe ratio is desirable for investors as it signifies a large return for little risk. We calculated the Sharpe ratio for our portfolios over one-year periods. In Exhibit 6, one can observe that the FT Global Equities Index consistently has a lower Sharpe ratio than the other strategies under consideration.

Recall that in the Capital Asset Pricing Model CAPM (see Merton [1992, chap. 16]), the index portfolio *is* the market portfolio in the sense of the Markowitz theory, and therefore should have the highest Sharpe ratio among efficient strategies, and, in particular, a higher Sharpe ratio than the minimum variance strategy. Exhibit 6 shows therefore that the FT Global Equities Index is *not* the market portfolio.

On the other hand, in Exhibit 7, one observes that the Sharpe ratio of the FT/US Index is outperformed by the minimum variance strategy in only 29% of the monthly periods, which again confirms the efficiency of the U.S. market.

The two remaining figures in the FT/US column of Exhibit 7 also warrant an explanation. For $p = -10$, the optimal risk adjusted returns strategy is defensive and should be on or near the capital market line of Exhibit 2, so its Sharpe ratio should be close to highest possible. This agrees with the fact that its Sharpe ratio was higher than that of the FT/US Index in all (= 100%) of the monthly periods considered. On the other hand, for $p = 0.5$, the optimal risk adjusted returns strategy has higher volatility and so it is to the upper right of the market portfolio: this lowers its Sharpe ratio, which is compatible with the 66% figure in Exhibit 7.

Finally, observe that the Sharpe ratio of the minimum variance strategy in the European market was higher than that of the FT Europe Index only 9% of the time. This is indicative of the fact that the expected return of the minimum variance strategy is rather low, and often not so much greater than the risk-free rate of return. In fact, the Sharpe ratio of the minimum variance strategy was negative in certain months.

3) *Risks and returns*. In any given time period, it is desirable to have a high return but also a low volatility. In Exhibit 8, we have plotted the return and volatility of the quantitative portfolios *relative to* the corresponding quantities for the FT Global Equities Index. That is, for each month from February 1994 to July 1999, and for each of the quantitative portfolios under consideration, we have plotted the point

$$\left(\frac{\text{volatility of portfolio}}{\text{volatility of the index}}, \frac{\text{return of the portfolio}}{\text{return of the index}} \right)$$

(see also formula (7) in the Appendix). This graph clearly shows that the minimum variance strategy consistently achieved higher returns with lower volatility than the index portfolio, while the risk adjusted returns strategies exhibit higher return but also higher volatility.

4) *Transactions*. A quantitative strategy that prescribes many small transactions is unattractive for most investors, both for reasons of convenience and because of transaction costs. This can also be the case of a strategy that frequently completely renews the contents of the portfolio. In Exhibit 9, the size of transactions is described by a representation of the portfolio turnover. Recall that the turnover is the percentage of the portfolio that is affected by rebalancing (see formula (8) in the Appendix). For each strategy, a "modified boxplot" representation of the monthly turnover is shown. The bottom, top and middle of the two boxes in the boxplot represent the second and third quartiles and the median of the monthly turnovers. The "whiskers" cover the 5th to 95th percentiles and all outliers are also plotted. Clearly, the typical monthly transaction for the minimum variance portfolio involves changing about 10% of the portfolio, while for the risk adjusted returns strategy (p = 0.5), there are many months with no transactions and several months in which the turnover is more than 40%. With p = -10, the turnover of the risk adjusted returns strategy compares favourably with the minimum variance strategy.

5) *Portfolio composition*. Exhibits 10 though 13 show the portfolio weightings and their evolution over the time period. In Exhibit 10, the vertical distance between two curves represents the weight of one sector in the Index. The sum of these weights is of course always equal to 1, so the top line is horizontal. Each weight varies over time, along with the value of that sector, since numbers of shares are held constant. The fact that these curves are largely parallel reflects the high degree of correlation between sectors. Indeed, during the time-period under consideration, these correlations are rarely negative and are generally greater than 0.8.

The weightings of the minimum variance strategy are shown in Exhibit 11. This Exhibit shows that the minimum variance strategy is well-diversified and largely without sudden large variations in portfolio weightings, as was also observed in Exhibit 9. Concerning the risk adjusted returns strategies, one observes immediately in Exhibits 12 and 13 that these strategies suffer from a lack of diversification among sectors. Indeed, with p = 0.5, the entire portfolio is generally invested in a single sector, which changes every few years. For short periods, the strategy invests in two sectors simultaneously. These rare but very large turnovers also show up in Exhibit 9. With p = -10, the diversification is greater, but it is rare that the strategy invests in more than four sectors. From May 1995 to January 1997, the portfolio is entirely invested in the two sectors Business Srvs. & Comp. Software and Aerospace/Defense Composite, which appear both in Exhibits 12 and 13 This lack of diversification is caused by the high correlation between sectors mentioned above. Indeed, one can show mathematically that when all correlations are sufficiently near 1, the risk adjusted returns strategy will pick out the sector with the highest return and invest exclusively in that sector, a rather unattractive feature from the point of view of achieving at least some diversification among sectors.

Conclusion

This empirical study has shown that mathematical models of financial markets can produce investment strategies with interesting characteristics. While the U.S. market largely fulfils theories of market efficiency, global markets do not and the quantitative investor is able to take advantage of this. Indeed, in global markets, over a substantial time period, the minimum variance strategy consistently achieved both higher returns and lower volatility than the benchmark. This also occurs in the European and Japanese markets, but not in the U.S. market. In global markets, passive strategies exhibit underperformance because available indices do not replicate the global market portfolio in the sense of the CAPM. Investors in global markets willing to accept higher volatility may achieve substantially higher returns than the benchmark or the minimum variance strategy by using the risk adjusted returns strategies, though these strategies suffer from a lack of diversification. The effect of transaction costs is difficult to quantify, but defensive strategies, including the minimum variance strategy, give rise to a turnover pattern that will be acceptable for most active investors.

References

Kallberg, J.G. and W.T. Ziemba. Misspecification in Portfolio Selection Problems, in: G. Bamberg and K. Spremann (Hrsg), *Risk and Capital*, New York: Springer (1984), pp. 74-87.

Kleeberg, Jochen M. Der Anlageerfolg des Minimum-Varianz Portfolios. Schriftenreihe "Portfoliomanagement". Uhlenbruch Verlag (1995).

Merton, Robert C. Optimum consumption and portfolio rules in a continuous-line model. *J. Economic Th.* 3 (1971), pp. 373-413.
Merton, Robert C. *Continuous-Time Finance*. Blackwell Publ. (1992).

Oksendal, Bernt. *Stochastic Differential Equations*. Second Edition. Springer-Verlag (1989).

Philips, Thomas K., Greg T. Rogers, and Robert E. Capaldi. Tactical Asset Allocation: 1977-1994. *The Journal of Portfolio Management* (1996), pp. 57-64.

Schäfer, K. and P. Zimmermann, Portfolio Selection und Schätzfehler bei den erwarteten Renditen, *Finanzmarkt und Portfolio Management* 12-2 (1998), pp. 131-149.

Appendix: mathematical formulas

The mathematical model. Let $S_i(t)$ denote the value of the i-th index at time t. We assume, in agreement with standard portfolio theory (see Merton [1992]), that the evolution of $S_i(t)$ is described by the stochastic differential equation

$$\mathrm{d}S_i(t) = S_i(t)\,(r_i(t)\,dt + \sum_{j=1}^{d} \sigma_{i,j}(t)\,\mathrm{d}B_j(t)), \quad i = 1, \ldots, d. \tag{1}$$

In this equation, $r_i(t)$ denotes the instantaneous return, $\sigma(t) = (\sigma_{i,j}(t))$ is the volatility matrix, and $B(t) = (B_1(t),...,B_d(t))$ is a d-dimensional Brownian motion. The functions $r_i(t)$ and $\sigma_{i,j}(t)$ are assumed to be deterministic and constitute the parameters of the model.

Maximising the risk adjusted return. Let $a(t) = (a_{i,j}(t))$ be the matrix defined by $a(t) = \sigma(t) \cdot \sigma^{\mathrm{T}}(t)$, set $w(t) = (w_1(t),..., w_d(t))^{\mathrm{T}}$, $r(t) = (r_1(t),..., r_d(t))^{\mathrm{T}}$, and fix a real number $p < 1$. Let $r_{\mathrm{f}}(t)$ denote the risk-free rate of return and let $\mathbf{1} = (1,...,1)^{\mathrm{T}}$. Set

$$\mathrm{J}(w(t)) = r_{\mathrm{f}}(t) + w^{\mathrm{T}}(t) \cdot (r(t) - r_{\mathrm{f}}(t)\mathbf{1}) + \frac{p-1}{2}\, w^{\mathrm{T}}(t) \cdot a(t) \cdot w(\mathrm{t}). \tag{2}$$

The constraints on the weights are

$$w_1(t) + ... + w_d(t) \leq 1, \quad w_i(t) \geq 0, \qquad i = 1, ..., d.$$

The proportion of capital invested in the risk-free asset is

$$1 - (w_1(t) + ... + w_d(t)).$$

A natural objective is to seek a strategy $(w(t), t \geq 0)$, which maximizes the quantity $\mathrm{J}(w(t))$ at each time t. This is a quadratic programming problem with linear constraints, and can be solved using standard software such as Matlab.

Minimising the volatility. Let $a(t)$ be as above, and set

$$\mathrm{V}(w(t)) = w^{\mathrm{T}}(\mathrm{t}) \cdot a(t) \cdot w(t). \tag{3}$$

The objective here is to seek a strategy $(w(t), t \geq 0)$ that minimises $\mathrm{V}(w(t))$ at each time t. We require that

$$w_1(t) + ... + w_d(t) = 1, \; w_i(t) \geq 0, \qquad i = 1, ..., d.$$

This is again a quadratic programming problem with linear constraints.

Estimating the parameters. Let $t_0, t_1,..., t_N$ be the first day of each of N+1 months. The value of each sector index is observed on these dates, which means that the data consists of the

values $S_i(t_n)$, $n = 0, ..., N$, $i = 1, ..., d$. In our study, we used data from January 1990 to July 1999, so $N = 114$, and recall that $d = 36$.

The estimator $\hat{r}_f(t_n)$ of the risk-free interest rate for the time period $]t_n, t_{n+1}]$ is simply the observed risk-free interest rate of the previous period: $\hat{r}_f(t_n) = r_f(t_{n-1})$. The observed rate $r_{i,n}$ of return per unit time (which we take equal to 1 year) of the sector index i over the time-period $]t_{n-1}, t_n]$ is given by

$$r_{i,n} = \frac{1}{t_n - t_{n-1}} \log\left(\frac{S_i(t_n)}{S_i(t_{n-1})}\right).$$

In order to estimate $r_i(t_n)$, we use a 48-month moving average ($m = 48$ was chosen so as to be significantly larger than the number $d = 36$ of sector indices, which is needed in order to consistently estimate the volatility matrix). Therefore, the estimator $\hat{r}_i(t_n)$ of $r_i(t_n)$ is

$$\hat{r}_i(t_n) = \text{mean}\,(r_{i,n-m+1},..., r_{i,n}), \qquad n = m,..., N. \tag{4}$$

Similarly, the volatility matrix $a(t_n)$ is estimated by

$$\hat{a}(t_n) = \text{Cov}\left[\begin{pmatrix} r_{1,n-m+1} \\ \vdots \\ r_{d,n-m+1} \end{pmatrix}, ..., \begin{pmatrix} r_{1,n} \\ \vdots \\ r_{d,n} \end{pmatrix}\right], \qquad n = m, ..., N. \tag{5}$$

Sharpe ratio. It is given by the formula

$$R(t) = \frac{r_p(t) - r_f(t)}{\sigma_p(t)} \tag{6}$$

where $r_p(t) = w^T(t) \cdot \hat{r}(t)$ and $\sigma_p^2(t) = w^T(t) \cdot \hat{a}(t) \cdot w(t)$.

Risks and returns. In Exhibit 8, for each month from February 1994 to July 1999, and for each of the quantitative portfolios under consideration, we have plotted the point

$$\left(\frac{\sigma_p(t)}{\sigma_I(t)}, \frac{r_p(t)}{r_I(t)}\right) \tag{7}$$

where $\sigma_p(t)$, $r_p(t)$ (respectively $\sigma_I(t)$, $r_I(t)$) denote the volatility and return of the quantitative portfolio (respectively the FT Global Equities Index).

Turnover. It is given by the formula

$$\text{turnover} = \frac{1}{2} \frac{\sum_i \left| w_i(t_n) S_i(t_n) - w_i(t_{n-1}) S_i(t_{n-1}) \right|}{\sum_i w_i(t_{n-1}) S_i(t_{n-1})}, \tag{8}$$

EXHIBIT 1
Global FT Sector Indices

Commercial Banks and Other Banks	Financial Institutions & Services
Insurance Life & Agents/Brokers	Insurance Multi/Property/Casualty
Real Estate	Diversified Holding Companies
Oil	Non-Oil Energy, Equip. & Srvs.
Utilities	Transportation and Storage
Automobiles	Household Durables & Appliances
Diversified Consumer Goods/Srvs.	Textiles and Wearing Apparel
Beverage Industries/Tobacco Mfg.	Health & Personal Care
Food & Grocery Products	Entertainment/Leisure/Toys
Media	Business Srvs. & Comp. Software
Retail Trade	Wholesale Trade
Aerospace/Defense Composite	Computer/Comm./Office Equip.
Electrical Equip.	Electronics & Instrumentation
Machinery & Engineering Srvs.	Auto Components
Diversified Industrials	Heavy Engineering & Shipbuilding
Construction & Building Materials	Chemicals
Mining, Metals & Minerals	Precious Metals and Minerals
Forestry and Paper Products	Fabricated Metal Products

EXHIBIT 2
The efficient frontier and the market portfolio in the Markowitz framework

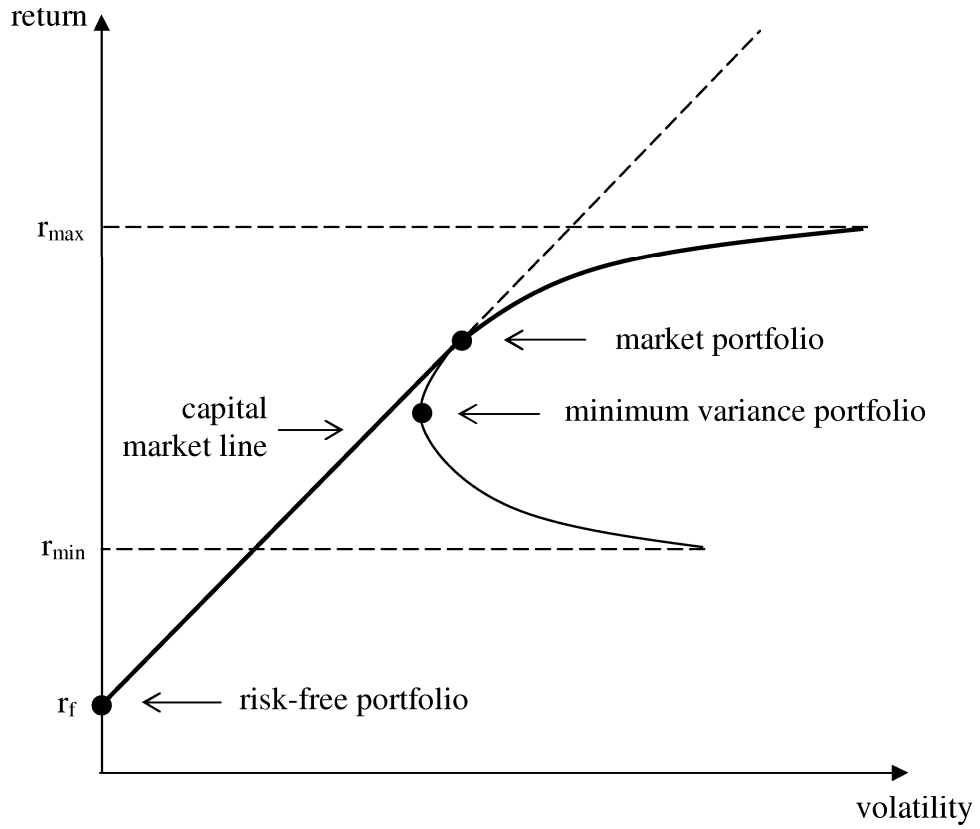

EXHIBIT 3
Performance (in US$) of the quantitative and index strategies in global markets

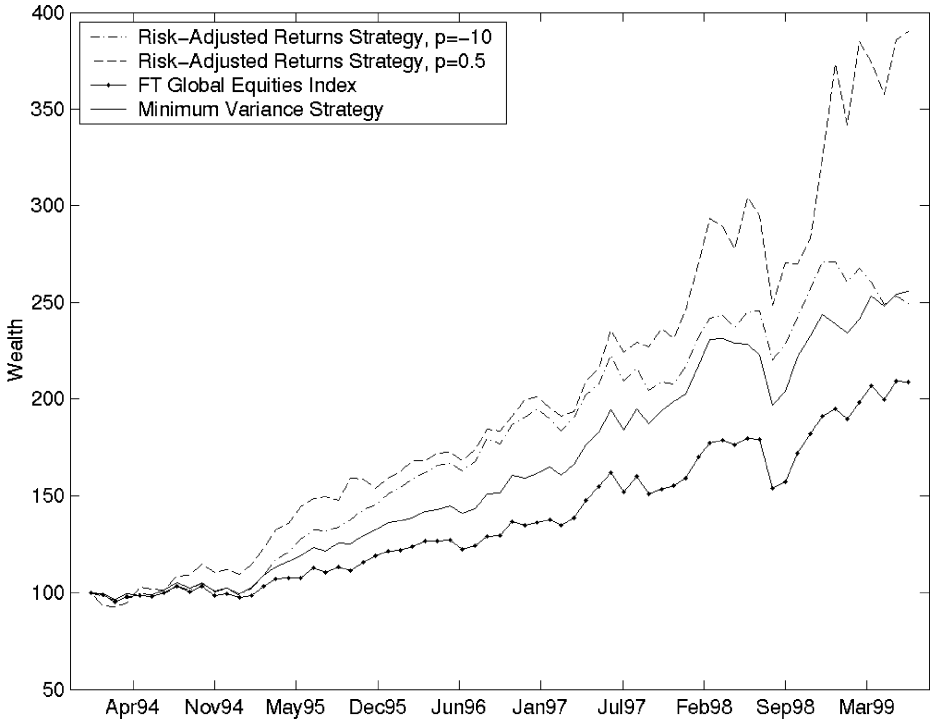

EXHIBIT 4
Performances, by geographic zones and currencies, of the quantitative and index strategies

	Global FT in US$	FT US in US$	FT Europe in US$	FT Europe in Euro	FT Japan in US$	FT Japan in J. Yen
Index	109%	156%	81%	62%	-15%	-24%
Minimum Variance	155%	94%	114%	111%	33%	27%
Risk-Adjusted Returns Strategy, p=-10	150%	161%	95%	17%	10%	2%
Risk-Adjusted Returns Strategy, p=0.5	290%	262%	79%	61%	30%	19%

EXHIBIT 5
Evolution of the volatilities (in US$) of the quantitative and index strategies in global markets

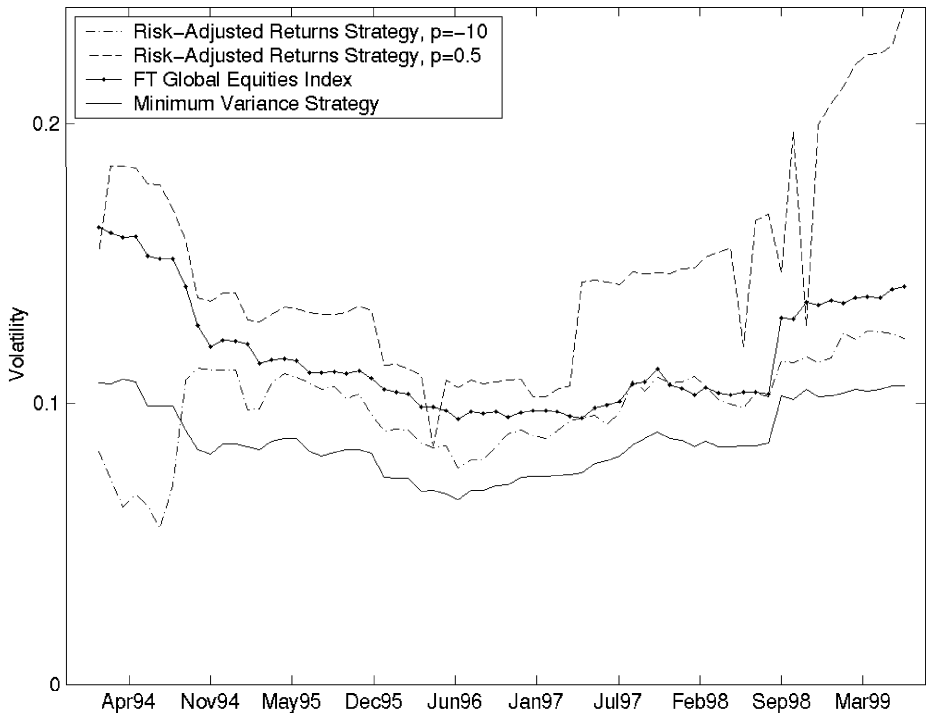

EXHIBIT 6
Evolution of the Sharpe Ratios (in US$) of the quantitative and index strategies in global markets

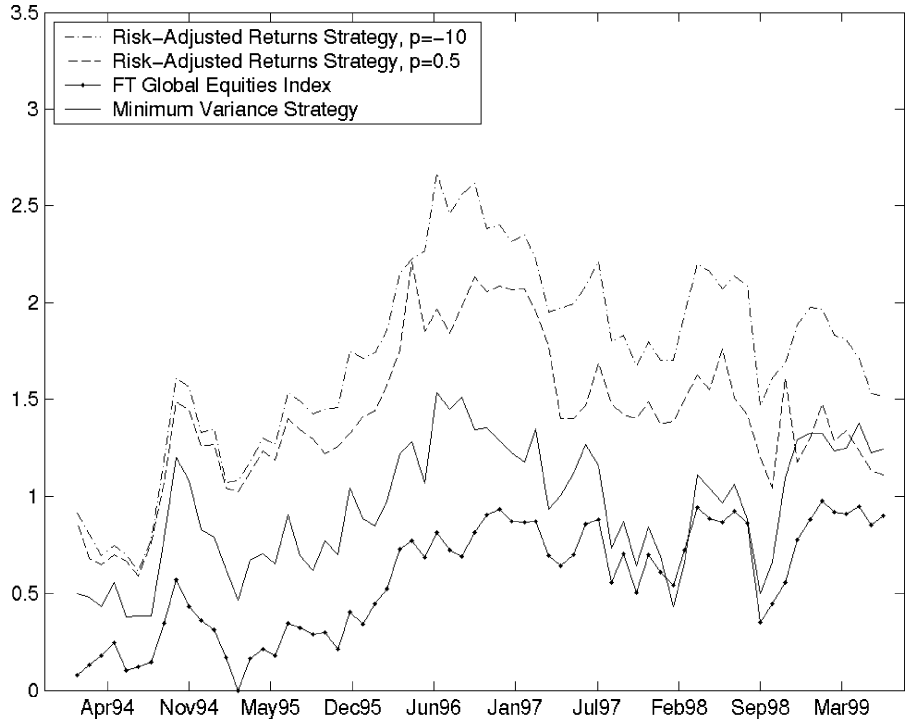

EXHIBIT 7
Percentages of months in which each quantitative strategy had a higher Sharpe ratio than the index strategy, by geographic zones and currencies

	Global FT in US$	FT US in US$	FT Europe in US$	FT Europe in Euro	FT Japan in US$	FT Japan in J. Yen
Minimum Variance	97%	29%	9%	9%	89%	89%
Risk-Adjusted Returns Strategy, p=-10	100%	100%	100%	100%	100%	100%
Risk-Adjusted Returns Strategy, p=0.5	100%	66%	100%	100%	100%	100%

EXHIBIT 8
Month by month relative volatilities and returns of the quantitative strategies, with respect to the index strategies (global markets in US$)

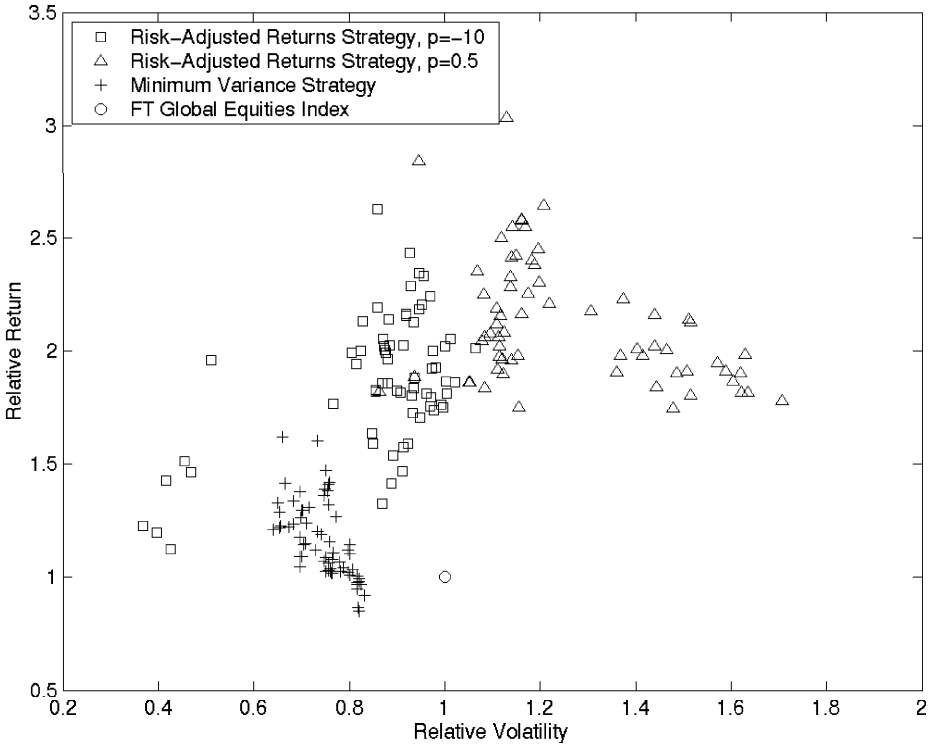

EXHIBIT 9
Boxplot of monthly turnovers of three quantitative strategies (global markets in US$)

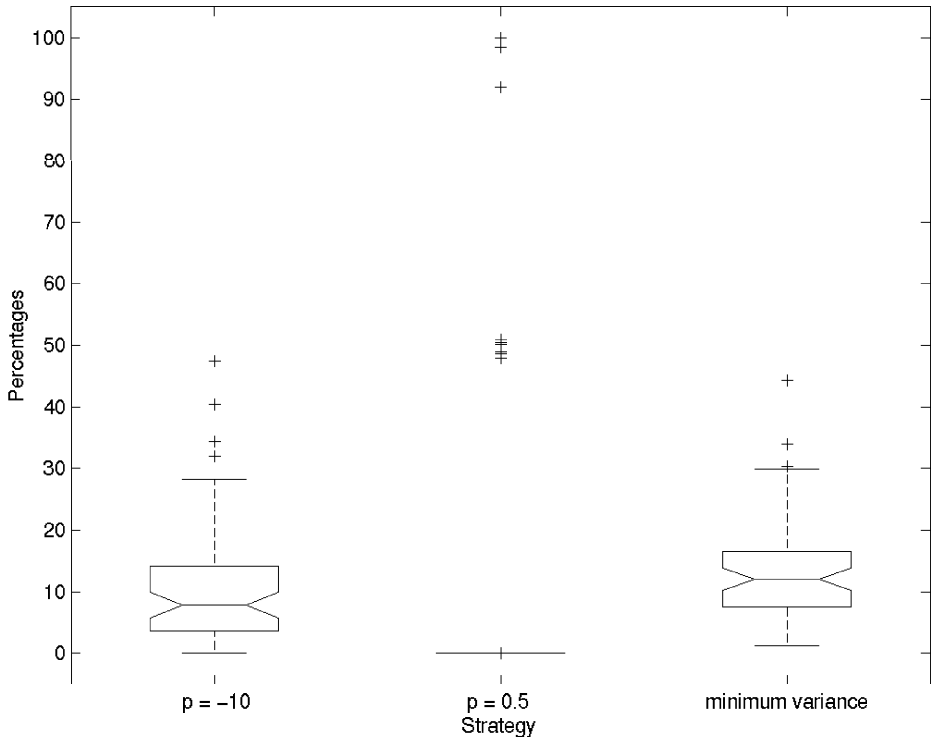

EXHIBIT 10
Evolution of the weightings of the FT Global Equities Index

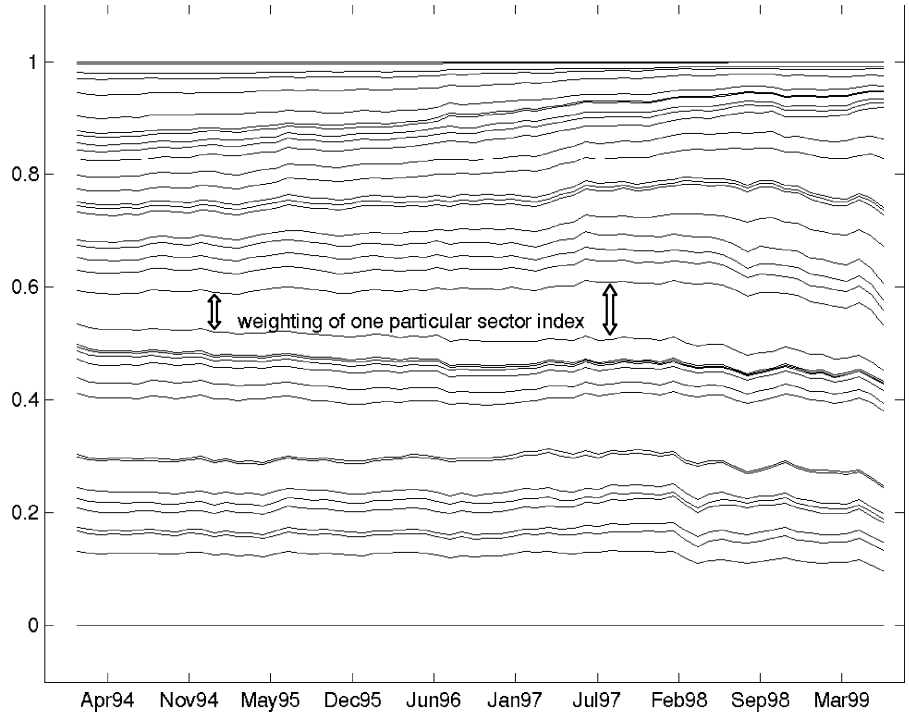

EXHIBIT 11
Evolution of the weightings of the minimum variance strategy (global markets in US$)

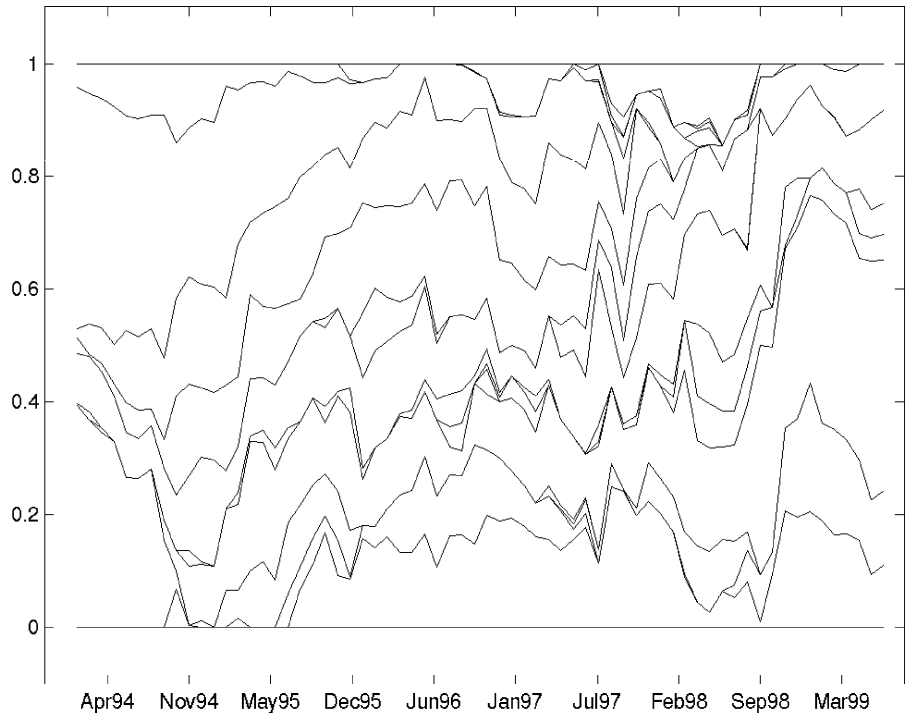

EXHIBIT 12
Evolution of the weightings of the risk-adjusted returns strategy (p=0.5, global markets in US$)

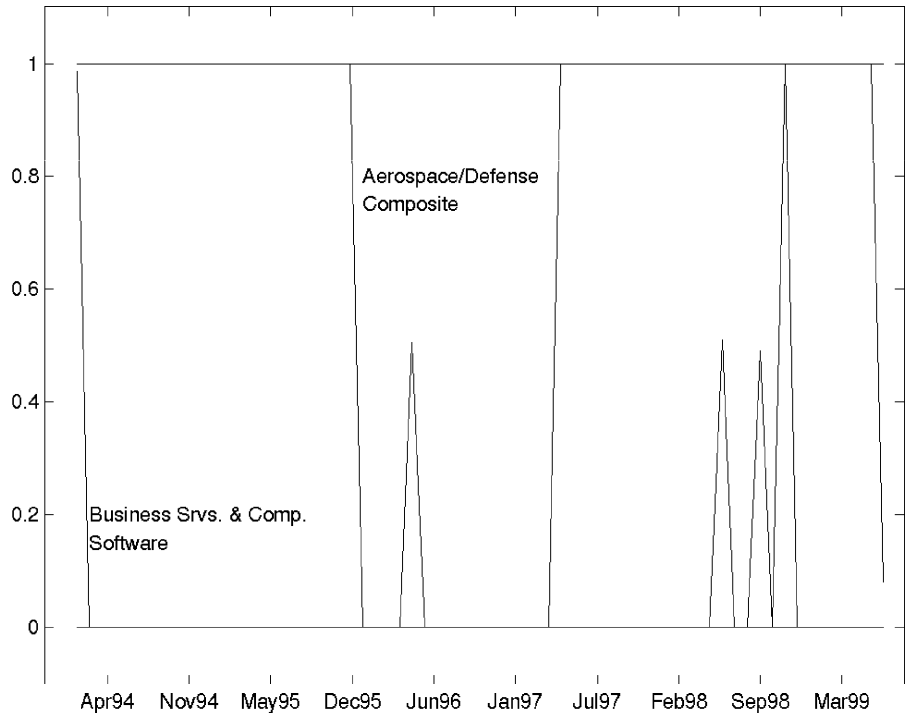

EXHIBIT 13
Evolution of the weightings of the risk-adjusted returns strategy (p=-10, global markets in US$)